

6.9.2019
Friday

Physics

Unit: 3 :- Kinematics :- Rest And Motion

- A body is said to be at rest if it doesn't change its position with respect to its surroundings.
- A body is said to be in motion if it changes its position with respect to the surrounding.

Displacement:-

- Displacement of a body is a vector connecting the initial and final positions of the body and is directed away from initial position towards the final position.
- Displacement is a vector quantity.
- Dimension of Displacement:- $[M^0 L^1 T^0]$
- S.I unit of Displacement:- Metre.

Speed:- Speed of a body is defined as the distance covered by the body in one second.
$$\text{Speed} = \frac{\text{distance}}{\text{Time}}$$

- * Speed is a scalar quantity.
- * If ' Δs ' is the distance covered by a body in time ' Δt ' then average speed = $\frac{\Delta s}{\Delta t}$.
- * Dimension of Speed = $[M^0 L^1 T^{-1}]$

Velocity:- Velocity of a body is defined as the rate of change of displacement.
$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

* Since displacement is a vector quantity, velocity is also a vector quantity.

* If $\Delta \vec{u}$ is the displacement of a body in time Δt then average velocity = $\frac{\Delta \vec{u}}{\Delta t}$.

* Dimensional Formula:- $[M^0 L^1 T^{-1}]$

* S.I unit :- $\frac{\text{Metre}}{\text{Second}}$

Acceleration:- * Acceleration is defined as the change in velocity in one second.

* Acceleration is a vector quantity.

* If $\Delta \vec{v}$ is the change in velocity of a body in a time interval Δt , then the average acceleration of the body is $\frac{\Delta \vec{v}}{\Delta t}$.

* Dimension:- $[M^0 L^1 T^{-2}]$.

* S.I unit:- M/sec^2

Acceleration in terms of displacement

We know that, $\vec{a} = \frac{d\vec{v}}{dt}$

But $\vec{v} = \frac{d\vec{u}}{dt}$

$\therefore \vec{a} = \frac{d}{dt} \left(\frac{d\vec{u}}{dt} \right) = \frac{d^2 \vec{u}}{dt^2}$

Thus, acceleration is also known as the second order derivative of displacement.

Force:- In Science, Force is the push or pull on an object with mass that causes it to change velocity (to accelerate).

$$F = m\vec{a}.$$

* Force is a vector quantity which means it has both magnitude and direction.

* From Newton's Second Law :- Force can also be defined as the rate of change of momentum.

$$F = \frac{d\vec{p}}{dt}$$

Where, Momentum, $\vec{p} = m\vec{v}$

* Dimension :- $[M^1 L^1 T^{-2}]$

* S.I. unit :- Newton

Newton :- $1 \text{ kg} \cdot \text{metre}/\text{sec}^2$.

Kinematics equation of motion:-

$$(i) S = ut + \frac{1}{2} at^2$$

$$(ii) v = u + at$$

$$(iii) v^2 - u^2 = 2as$$

Where, S = Displacement of the body

u = Initial velocity of the body.

v = Final velocity of the body.

a = Acceleration of the body

t = Time taken by the body.

Equation of Motion Under Gravity:-

g = Acceleration due to gravity

Dimension of g = $[M^0 L T^{-2}]$

5. Unit of $g = 1/Sec^2$

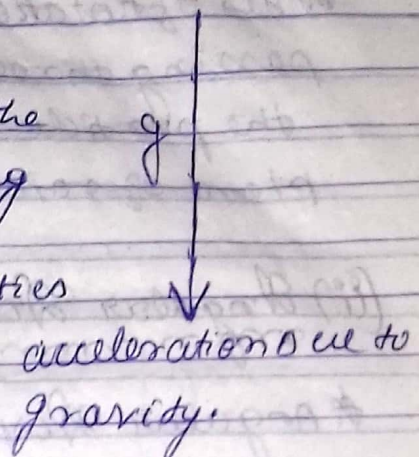
a) Downward motion:-

If a body moves freely under the action of gravity corresponding eqn's of motion can be obtained by replacing a with g in kinematics eqn of motion.

(i) $s = ut + \frac{1}{2}gt^2$

(ii) $v = u + at$

(iii) $v^2 - u^2 = 2gs$



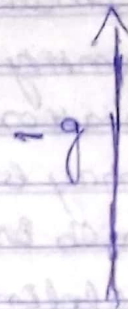
b) Upward motion:-

* Here a is replaced by $-g$.

1) $s = ut - \frac{1}{2}gt^2$

2) $v = u - gt$

3) $v^2 - u^2 = -2gs$



Circular Motion:-

* The body is said to move in circular motion if it moves in such a way that its distance from a fixed point always remains constant.

* The path of the body is called a circle. The fixed point is called the centre of the circle & the fixed distance is called radius of the circle.

Examples of circular motion:-

(i) The motion of planet around the sun.

(ii) The motion of electron around the nucleus.

(iii) A car or a cyclist taking a turn.

(d) Axes of Rotation:-

Axis of rotation of a particle is a straight line passing through the centre of the circle in which the particle rotates and is perpendicular to the plane of rotation.

(ii) Angular Displacement $\vec{\theta} = (\Delta \theta) \hat{e}$

* Angular Displacement of a particle undergoing rotational motion is defined as the angle turned by it + radius vector.

* If small angular displacement is a vector quantity then it is directed along axis of rotation.

* It may be directed upward or downward which is given by the right hand thumb Rule.

* Holding the axis of rotation in right hand if the finger falls in the direction of the rotation of the body then thumb gives the direction of small angular displacement.

Imp Relation Between Linear Displacement ($\Delta \vec{s}$) And Angular Displacement

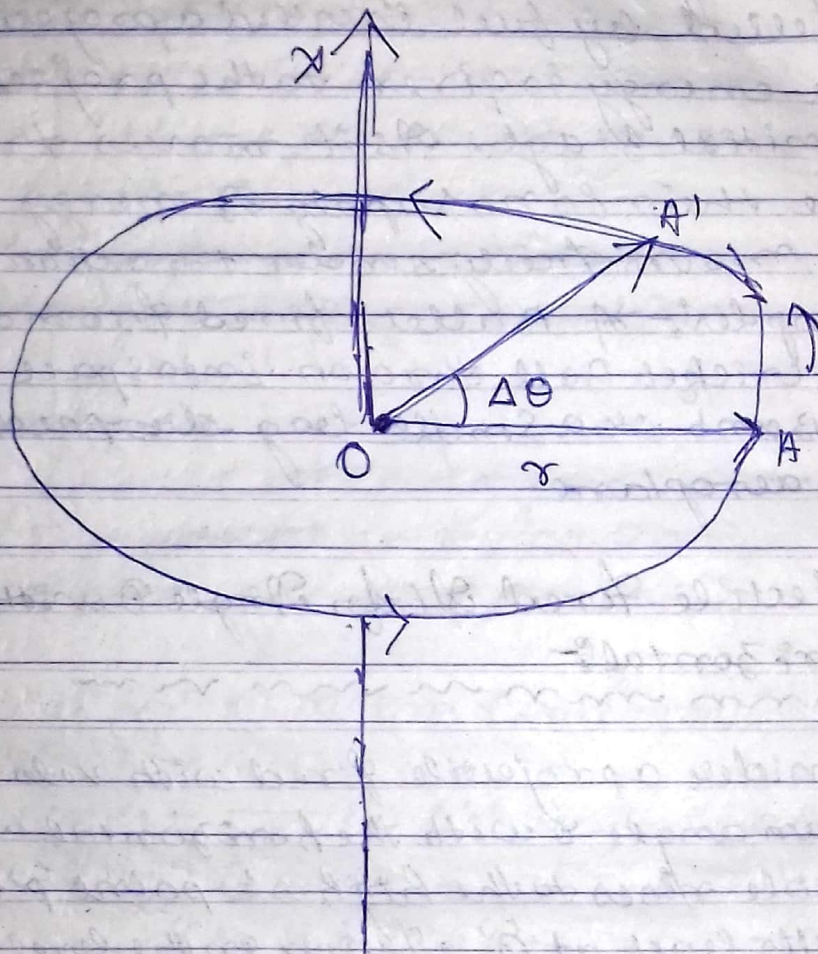
(c) Scalar Form - Consider a body undergoing uniform circular motion (motion in a circle with uniform speed in the plane of the paper).

Let Δs be the linear displacement (along the curve) corresponding to the displacement $\Delta \theta$

$$\Delta \theta = \frac{\Delta s}{r}$$

$$\Rightarrow \Delta s = r \Delta \theta$$

Linear Displacement = Radius $\times$ Angular Displacement.



(ii) Vector Form:-

$$\Delta \vec{s} = \Delta \vec{\theta} \times \vec{r}$$

Unit of Angular Displacement:-

radian (rad).

(ii) Angular Velocity:- ($\vec{\omega}$)

Angular velocity of a particle undergoing rotational motion is defined as the rate of change of angular displacement with time.

$$\therefore \vec{\omega} = \frac{\Delta \vec{\theta}}{\Delta t}$$

Projectile Motion:-

* Body projected into space and is no longer propelled by fuel is called a projectile.

* Some energy is given to the projectile at the initial stage. As it moves through space there is no supply of energy to it and it moves freely under the action of gravity.

Examples - A Bullet fired from a rifle.

→ A Cricket Ball thrown into space

→ A Bomb or a small bag dropped from an aeroplane.

V.V.gmp

Projectile Fired At an Angle θ with The Horizontal

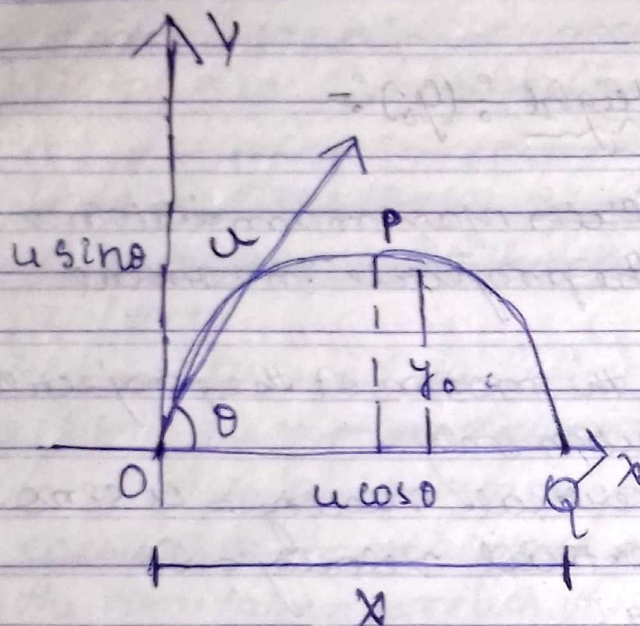
* Consider a projectile fired with velocity u at an angle θ with the horizontal. The projectile rises to the highest point P and falls back at Q . Lying on the level of projection. Resolving it in two components $u \cos \theta$ along horizontal this component is uniform since g has no component along this direction.

* Along vertical upward direction this component is non-uniform since g act exactly opposite to it.

* Above mentioned two components are independent of each other since they're mutually perpendicular.

* Following important results can be derived in this case:-

(C) Eqn of Trajectory:-



Horizontal eqⁿ of motion (uniform) :-

$x = \text{Velocity} \times \text{time}$

$$= (u \cos \theta) t \quad \text{--- (i) } \because s = ut + \frac{1}{2} at^2$$

Vertical eqⁿ of motion,

$$y = (u \sin \theta) t - \frac{1}{2} g t^2 \quad \text{--- (ii)}$$

Here, $u \sin \theta$ is the initial velocity of the projectile in the vertical direction and acceleration is taken to be negative since its direction is opposite (downward) to that of the velocity (upward).

From Eqⁿ (i)

$$t = \frac{x}{u \cos \theta} \quad \text{--- (iii)}$$

Substituting eqⁿ (iii) in eqⁿ (ii), we have,

$$y = u \sin \theta \left(\frac{x}{u \cos \theta} \right) - \frac{1}{2} g \left(\frac{x}{u \cos \theta} \right)^2$$

$$= x \tan \theta - \frac{g}{2 u^2 \cos^2 \theta} x^2 \quad \text{--- (iv)}$$

This is the eqⁿ of a parabola. Hence the trajectory of a projectile is parabolic.

(ii) Maximum Height :- (y_0) :-

It is the ~~maximum~~ maximum distance travelled by the projectile in vertical direction.

Let us consider the motion of the projectile in vertical direction only.

At O', initial vertical velocity = $u \sin \theta$.

At P', final vertical velocity = 0.

Acceleration = $-g$.

Vertical distance travelled = y_0 .

Applying kinematic Relation,
 $v^2 - u^2 = 2as$

$$\Rightarrow 0 - (u \sin \theta)^2 = 2(-g)y_0$$

$$\Rightarrow y_0 = \frac{u^2 \sin^2 \theta}{2g} \quad (v)$$

(iii) Time of ascent (t) :-

It is the time taken by the projectile to rise ^{to} ~~from~~ the ~~projectile~~ highest point.

Again considering motion in vertical direction only.

At O', initial vertical velocity = $u \sin \theta$.

At P', final vertical velocity = 0.

Acceleration = $-g$.

Time = t .

Applying kinematic's relation,
 $v = u + at$

$$\Rightarrow 0 = u \sin \theta - gt$$

$$\Rightarrow gt = u \sin \theta$$

$$\Rightarrow t = \frac{u \sin \theta}{g} \quad \text{--- (vi)}$$

(iv) Total time of flight (T):

It is the time taken by the projectile to come back to the same level from which it was projected.

It can be derived in following two ways:-

a) Since the motion from O' to P' and that from P' to Q' is symmetric with respect to each other, the total time of flight is twice the time taken to reach the highest point.

$$T = 2t$$

Substituting t from eqⁿ (vi),

$$T = \frac{2u \sin \theta}{g} \quad \text{--- (vii)}$$

b) Expression for T can also be derived independently.

Considering motion in vertical direction only,

At O', Initial vertical velocity = $u \sin \theta$

Vertical acceleration = $-g$.

Time to travel from O to Q = T

Vertical displacement betⁿ O and Q = 0.

Applying kinematic relⁿ:-

$$h = ut + \frac{1}{2} at^2$$

$$\Rightarrow 0 = (u \sin \theta)T + \frac{1}{2}(-g)T^2$$

$$\Rightarrow (u \sin \theta)T = \frac{1}{2}gT^2$$

$$\Rightarrow T = \frac{2u \sin \theta}{g}$$

(v) Horizontal Range (X):-

It is the distance travelled by the projectile in the horizontal direction.

$$X = \text{Horizontal velocity} \times \text{Total time of flight}$$

$$= u \cos \theta \times T$$

$$= u \cos \theta \times \frac{2u \sin \theta}{g}$$

$$= \frac{u^2 2 \sin \theta \cos \theta}{g}$$

$$= \frac{u^2 \sin 2\theta}{g} \quad \text{--- (vii)}$$

Condition for Maximum Horizontal Range

From eqn (vii), it is clear that the horizontal range depends on u and θ . For a fixed u , the range can be changed by changing θ . Range will be maximum if $\sin 2\theta$ will be maximum i.e., $\theta = 45^\circ$ which is the condition for maximum horizontal range.