Simple Machine

Simple Machine - A simple machine may be defined as a device, which enables us to do some useful work at some point or to overcome some resistance, when an effort / force is applied on it.

or

Simple machine is such a device, which requires some force to do some ~~wo~~ useful work.
Ex:- pulley

Compound machine.
When a number of simple machines are combined together to do some useful work at a faster speed, then it is called compound machine.

Lifting machine.
It is a device, which enables us to lift a heavy load (W) by applying a comparatively less effort (P).

Mechanical Advantage (M.A)-
The mechanical advantage is the ratio of weight lifted (W) to the effort applied (P).
Mathematically,

$$M.A = \frac{W}{P}$$

Efficiency of a machine-
It is the ratio of output to the input of a machine and is generally expressed in percentage.

$$\eta = \frac{\text{output}}{\text{Input}} \times 100$$

Ideal Machine -

The machine which has 100% efficiency.

Velocity Ratio -

It is the ratio of a distance through which any part of a machine moves, to that which the driving part moves during the same time.

or

$$V.R = \frac{\text{dist. moved by the point at which effort is applied.}}{\text{dist. moved by the point at which load is applied.}}$$

Note. In case of ideal machine,
V.R = M.A

Relation betn efficiency, mechanical advantage and velocity ratio of a lifting machine.

Let us take,

W = load lifted by the machine
P = Effort required to lift the load.
y = distance moved by the effort
x = Distance moved by the load.

$$M.A = \frac{W}{P}$$

$$V.R = \frac{y}{x}$$

We know that,

input of a machine = work done on the machine

output " " = " by the machine.

In a lifting machine, input is measured by the product of effort and the distance through which it has moved, and output is measured by the product of weight lifted and the distance through which it has been lifted.

$$\therefore [\text{Input}]_{\text{simple machine}} = \text{Effort} \times \text{distance moved}$$

$$= P \times y$$

$$[\text{output}]_{\text{simple machine}} = \text{weight lifted} \times \text{distance moved}$$

$$= (W) \times (x)$$

$$\eta = \text{efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{W \times x}{P \times y} = \left(\frac{W}{P}\right) \times \left(\frac{x}{y}\right)$$

$$= (M.A) \times \left(\frac{1}{V.R}\right)$$

$$= \frac{M.A}{V.R}$$

So, $$\boxed{\eta = \frac{M.A}{V.R}}$$

<u>Note.</u> In case of ideal machine $\eta = 1$ (100% efficiency)

So, M.A = V.R

Q/ In a certain weight lifting machine, a weight of 1 kN is lifted by an effort of 25 N. While the weight moves up by 100mm, the point of application of effort moves by 8 m. Find the mechanical advantage, V.R and η of machine

Reversibility of a machine.

A machine is called reversible machine, when it does work in the reversed direction, after the effort is removed.

Condition for the reversibility of a machine.

If W = load lifted by the machine
P = effort required
y = Distance moved by the effort
x = Distance moved by the load.

Input = $P \times y$
output = $W \times x$

machine friction (loss) = Input - output
$= (P \times y) - (W \times x)$

For reversible machine,
output > machine friction

$\Rightarrow W \times x > (P \times y) - (W \times x)$

$\Rightarrow 2(W \times x) > P \times y$

$\Rightarrow \dfrac{W \times x}{P \times y} > \dfrac{1}{2}$

$\Rightarrow \dfrac{(W/P)}{(y/x)} > \dfrac{1}{2} \Rightarrow \dfrac{M.A}{V.R} > \dfrac{1}{2}$

$\Rightarrow \eta > \dfrac{1}{2} \Rightarrow \eta > 0.5$

$\Rightarrow \boxed{\eta > 50\%}$

Hence, the condition for a machine to be reversible, is efficiency should be more than 50%.. ($\eta > 50\%$)

Self-locking Machine.

- The machine is said to be self-locking if the load is not lowered on removal of the effort.
- A lifting machine is self locking if its efficiency is less than 50%..

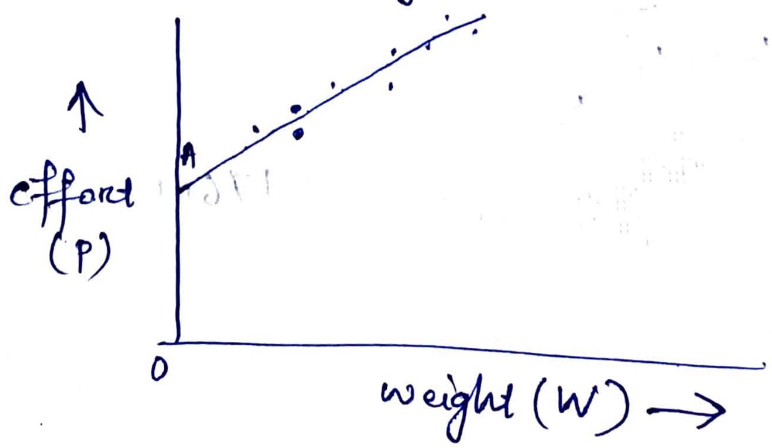
Note:

Reversible machine = removal of effort results in lowering of the load = $\eta > 50\%$.

Irreversible machine = removal of effort doesn't lower the load = $\eta < 50\%$

Law of Machine -

- The relationship betⁿ the load lifted and the effort required in a machine is called the law of machine.
- This is generally a straight line, which doesn't pass through the origin.
- This is found by conducting experiments
- the load vs effort graph is shown below



OA = amount of friction offered by the machine.

Mathematically,

$$P = (m \times W) + C$$

P = effort applied to lift the load.

m = a constant (coeff of friction)
 = slope of line AB.

W = load lifted.

C = Another constant, which represents the machine friction.

Friction of M/c in terms of effort = $F_{effort} = P - \frac{W}{V.R}$

Q/ What load can be lifted by an effort of 120N, if the V.R = 18 and efficiency = 60%.

Q/ In a lifting machine, an effort of 40N raised a load of 1KN. If efficiency of the machine is 0.5, what is its V.R?
- If on this machine, an effort of 74N raised a load of 2KN, what is now the efficiency?
- What will be the effort required to raise a load of 5KN?

Ans_ V.R = 50
$\eta = 54\%$.
C = 6, M = 0.034, P = 176N

Q/ What load will be lifted by an effort of 12N, if the velocity ratio is 18 and efficiency of the machine at this load is 60%.
If the m/c has a const friction resistance, determine the law of machine and find the effort required to run this machine at (i) no load (ii) a load of 900N.

Ans/ $MA = \frac{load}{effort} = \frac{load}{12} = \frac{W}{12}$

$\eta = \frac{MA}{VR} \Rightarrow 0.6 = \frac{MA}{18} \Rightarrow MA = 18 \times 0.6$

$\Rightarrow \frac{W}{12} = MA \Rightarrow W = MA \times 12 = 18 \times 0.6 \times 12 = 129.6N$

law of machine.

$P = mW + C$

effort ← P, slope ← m, load ← W, C → machine friction (const)

$C = P - \frac{W}{V.R} = 12 - \frac{129.6}{18} = 4.8N$

$P = (m \times W) + 4.8$ —— (1)

putting $P = 12N$, $12 = (m \times 129.6) + 4.8$

$\Rightarrow m = \frac{1}{18}$

Law of machine: $P = \frac{1}{18}W + 4.8$

with no load, $W = 0$, $P = 4.8$

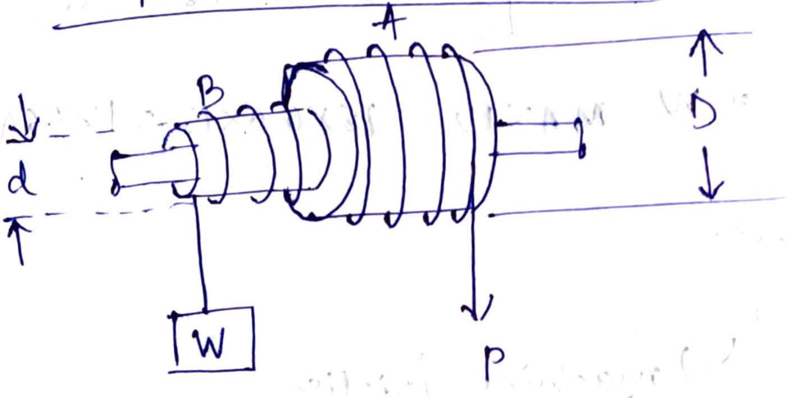
$W = 900N$, $P = \left(\frac{1}{18} \times 900\right) + 4.8 = 54.8$

Simple Lifting Machine

important lifting machines

1. Simple wheel and axle
2. Worm and Worm wheel
3. Single purchase crab winch
4. Double purchase crab winch
5. Simple screw jack

Simple wheel and axle



A = wheel
B = Axle

- A string is wound round the axle B, which carries the load to be lifted.
- A second string is wound round the wheel A in opposite direction to that of string on B.

D = dia of effort wheel
P = effort applied to lift the load.
W = load lifted
d = dia of load axle

A downward motion of effort (P) will raise the load (W).

when wheel rotates 1 revolution, the axle will rotate through 1 revolution.

displacement of effort in 1 revolution $= \pi D$

" " load " " " $= \pi d$

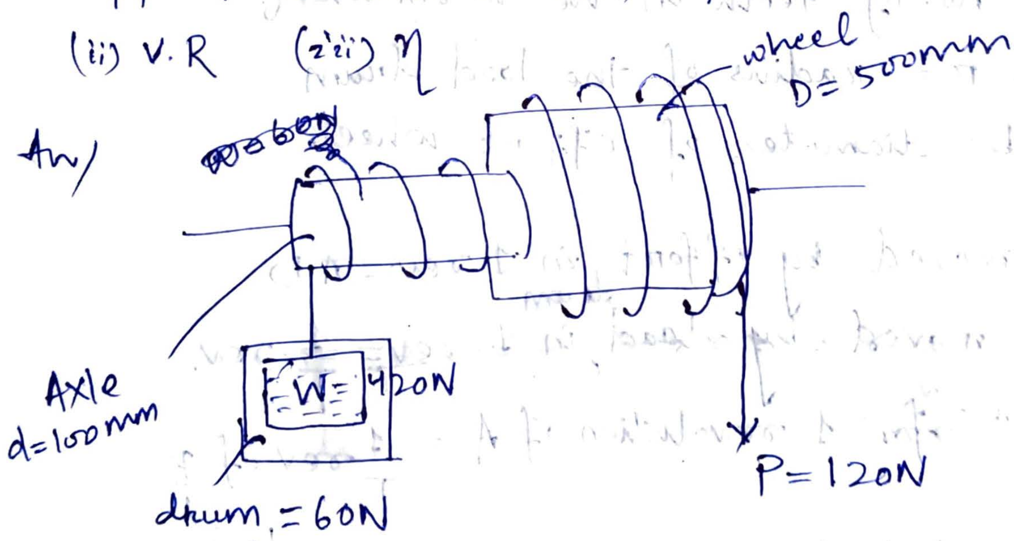
$$V.R = \frac{\text{dist moved by the effort}}{\text{dist moved by the load}} = \frac{\pi D}{\pi d} = \frac{D}{d}$$

$$M.A = \frac{\text{load lifted}}{\text{Effort applied}} = \frac{W}{P}$$

$$\eta = \frac{M.A}{V.R}$$

Q/ A drum weighing 60N and holding 420N of water is to be raised from a well by means of wheel and axle. The axle is 100mm diameter and the wheel is 500mm diameter. If a force of 120N has to be applied to the wheel, find (i) mechanical adv. (ii) V.R (iii) η

Ans/

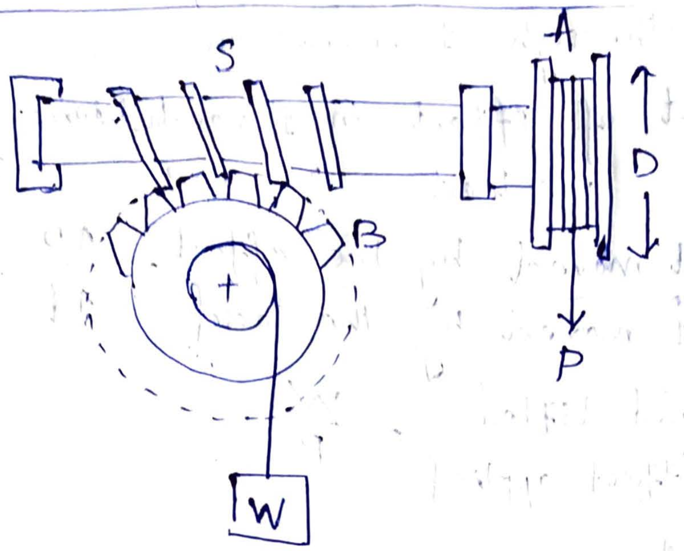


$$MA = \frac{\text{load}}{\text{effort}} = \frac{(60+420)N}{120N} = \frac{480}{120} = 4$$

$$V.R = \frac{\text{dist moved by effort}}{\text{dist moved by load}} = \frac{\pi D}{\pi d} = \frac{D}{d} = \frac{500}{100} = 5$$

$$\eta = \frac{MA}{V.R} = \frac{4}{5} = 0.8 = 80\%$$

Worm and Worm Wheel



S = square single threaded screw (Known as worm)

B = toothed wheel (worm wheel)

A = wheel attached to worm

W = load lifted

P = effort applied to lift the load

T = no. of teeth on the worm wheel

r = radius of the load drum

D = diameter of effort wheel

dist moved by effort in 1 rev = πD

for 1 revolution of A = $\frac{1}{T}$ rev of B

dist through which load will move (load drum) = $\frac{2\pi r}{T}$

$$V.R = \frac{\text{dist moved by effort}}{\text{dist moved by load}}$$

$$= \frac{\pi D}{\left(\frac{2\pi r}{T}\right)} = \frac{DT}{2r}$$

$$\boxed{V.R = \frac{DT}{2r}}$$

$$M.A = \frac{W}{P}$$

$$\eta = \frac{M.A}{V.R}$$

note: if the worm is double threaded

$$V.R = \frac{DT}{4r}$$

If the worm is 'n' threaded,

$$V.R = \frac{DT}{2nr}$$

Q/ A worm and worm wheel with 40 teeth on the worm wheel has effort wheel of 300mm dia and load drum of 100mm dia. Find η of machine, if it can lift a load of 1800 N with an effort of 24N.

Ans/ D = 300 mm | T = 40
r = 50mm

$$VR = \frac{DT}{2r} = \frac{300 \times 40}{2 \times 50} = 120$$

$$MA = \frac{Load}{effort} = \frac{1800}{24} = 75$$

$$\eta = \frac{MA}{VR} = \frac{75}{120} = 0.625 = 62.5\%$$

Q/ In a double threaded worm and worm wheel the no. of teeth on the worm wheel is 60. The diameter of the effort wheel is 250mm. and that of load drum is 100mm. Calculate the velocity ratio. If $\eta = 50\%$, determine effort required to lift a load of 300N.

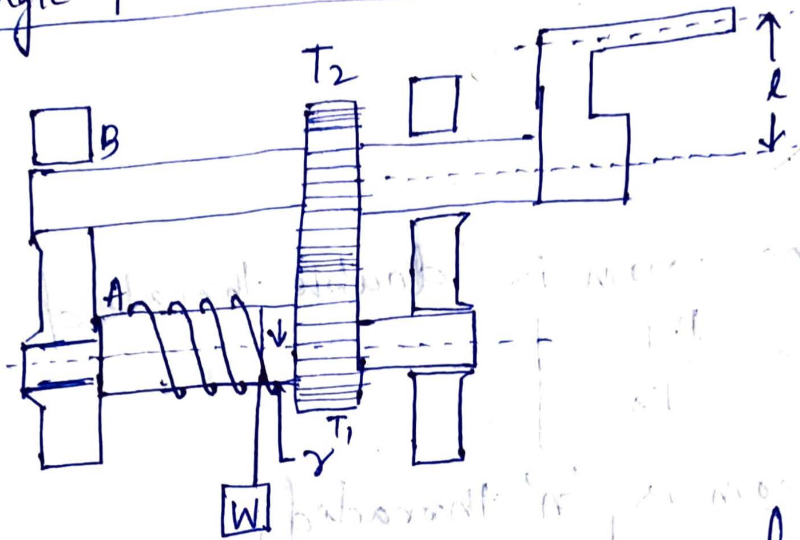
Ans/ T = 60
D = 250mm
r = 50mm

$$V.R = \frac{DT}{4r} = \frac{250 \times 60}{4 \times 50} = 75$$

$$\eta = \frac{MA}{VR} \Rightarrow MA = \eta \times VR = 0.5 \times 75 = 37.5$$

$$MA = \frac{Load}{effort}, \quad effort = \frac{300}{37.5} = 8$$

Single purchase crab winch.



A rope is fixed to the drum and is wound round it. The free end of the rope carries the load W.

A toothed wheel A is rigidly mounted on the load drum. Another toothed wheel B, called pinion, is geared with the toothed wheel A.

The effort is applied at the end of the handle to rotate it.

T_1 = no. of teeth on the main gear A.

T_2 = no. of teeth on the pinion B.

l = length of the handle.

r = Radius of load drum

W = load lifted

P = effort applied to lift the load.

Distance moved by the effort in one revolution of the handle $= 2\pi l$

no. of revolutions made by the pinion B = 1

no. of revolutions " " " wheel A $= \frac{T_2}{T_1}$

no. of revolutions made by the load drum $= \frac{T_2}{T_1}$

and distance moved by the load $= 2\pi r \times \frac{T_2}{T_1}$

$$V.R = \frac{\text{dist moved by effort}}{\text{dist moved by load}} = \frac{2\pi l}{2\pi r \times \frac{T_2}{T_1}} = \frac{l}{r} \times \frac{T_1}{T_2}$$

$$\boxed{V.R = \frac{l}{r} \times \frac{T_1}{T_2}}$$

$$M.A = \frac{W}{P}$$

$$\eta = \frac{M.A}{V.R}$$

Q/ In a single purchase crab winch, the no. of teeth on pinion is 25 and that on the spur wheel 100. Radii of the drum and handle are 50 mm and 300mm respectively. Find the efficiency of the machine and the effect of friction, if an effort of 20N can lift a load of 300N.

Ans)
$$V.R = \frac{300}{50} \times \frac{100}{25} = 24$$

$$M.A = \frac{W}{P} = \frac{300}{20} = 15$$

$$\eta = \frac{MA}{VR} = 62.5\ \%$$

effect of friction in terms of effort,

$$(F)_{effort} = P - \frac{W}{V.R} = 20 - \frac{300}{24} = 7.5\ N$$

Q/ A single purchase crab winch, has the following details :

Length of lever = 700 mm
no. of pinion teeth = 12
no. of spur gear teeth = 96
dia of load axle = 200mm

It is observed that an effort of 60N can lift a load of 1800N and an effort of 120N can lift a load of 3960N.
What is law of machine? Also find η of the machine in both the cases.

$l = 700\ mm$

$T_2 = 12,\ T_1 = 96$

$r = 100\ mm$

when $P_1 = 60N,\ W_1 = 1800N,\ P_2 = 120N,$

$W_2 = 3960N$

$P = mW + C$

$$\begin{bmatrix} 60 = (m \times 1800) + C \\ 120 = (m \times 3960) + C \end{bmatrix}$$

$m = \frac{1}{36}$

$C = 10$

$P = \left(\frac{1}{36} \times W\right) + 10$

$V.R = \frac{l}{r} \times \frac{T_1}{T_2} = \frac{700}{100} \times \frac{96}{12} = 56$

$M.A = \frac{W_1}{P_1} = \frac{1800}{60} = 30$

$\eta_1 = \frac{M.A}{V.R} = \frac{30}{56} = 0.536 = 53.6\%$

$M.A]_{2nd\ case} = \frac{W_2}{P_2} = \frac{3960}{120} = 33$

$\eta_2 = \frac{MA}{VR} = \frac{33}{56} = 0.589 = 58.9\%$

Double Purchase crab winch.

It is an improved form of a single purchase crab winch, in which the velocity ratio is intensified with the help of one more spur wheel and a pinion.

- In this there are 2 spur wheels of teeth T_1 and T_2 and T_3 as well as 2 pinions of teeth T_2 and T_4.

T_1 and T_3 = no. of teeth of spur wheels

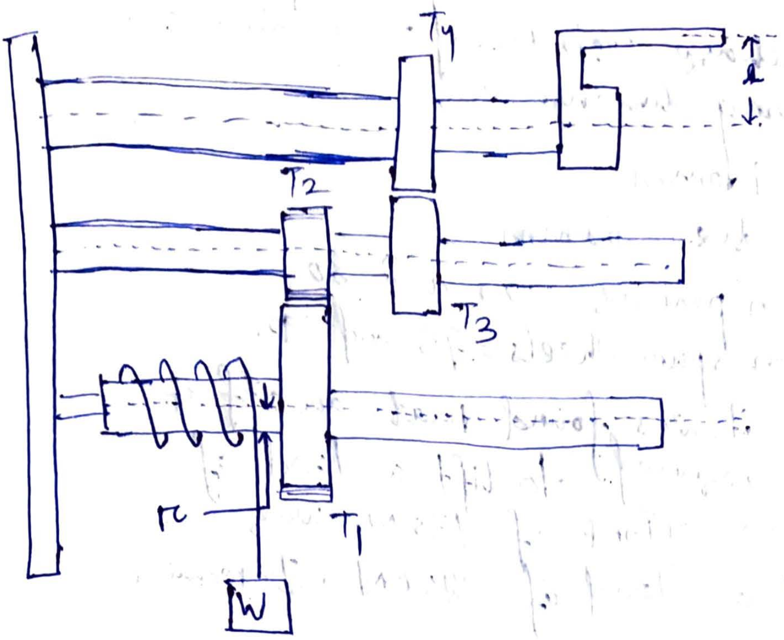
T_2 and T_4 = " " of pinions.

l = length of handle

r = rad of load drum

W = load lifted

P = effort applied



$= \frac{T_2}{T_1}$

$\frac{T_2 \times \frac{T_4}{T_3}}{T_1}$

dist moved by effort in 1 rev. of handle $= 2\pi l$

∴ no. of revolutions made by pinion 4 $= 1$

no. of revolutions made by wheel 3 $= \frac{T_4}{T_3}$

no. of revolutions made by pinion 2 $= \frac{T_4}{T_3}$

" " " wheel 1 $= \frac{T_2}{T_1} \times \frac{T_4}{T_3}$

dist moved by the load $= 2\pi r \times \frac{T_2}{T_1} \times \frac{T_4}{T_3}$

$$V.R = \frac{\text{Dist. moved by the effort}}{\text{Dist moved by the load}}$$

$$= \frac{2\pi l}{2\pi r \times \frac{T_2}{T_1} \times \frac{T_4}{T_3}} = \frac{l}{r}\left(\frac{T_1}{T_2} \times \frac{T_3}{T_4}\right)$$

$$M.A = \frac{W}{P}$$

$$\eta = \frac{M.A}{V.R}$$

Q) In a double purchase crab winch, teeth of pinions are 20 and 25 and that of spur wheels are 50 and 60. length of handle is 0.5 m and rad of load drum is 0.25 m. If efficiency of the m/c is 60%. Find the effort reqd to lift a load of 720N.

Ans) $VR = \frac{l}{r}\left(\frac{T_1}{T_2} \times \frac{T_3}{T_4}\right) = \frac{0.5}{0.25}\left(\frac{50}{20} \times \frac{60}{25}\right) = 12$

$MA = \frac{720}{P}$

$\eta = \frac{MA}{V.R} = \frac{60}{P}$, $P = 100N$

Q/ A double purchase crab used in a laboratory has the following dimensions.
dia of drum = 180mm
length of handle = 360 mm
no. of teeth on pinions = 20 and 30
" " on spur wheels = 75 and 90.
When tested it was found that an effort of 90N was required to lift a load of 1800N and an effort of 135N was reqd to lift a load of 3150N. Determine
(i) Law of m/c
(ii) probable effort to lift a load of 4500N
(iii) η
(iv) η_{max}.

Ans/ $P = mW + C$

$90 = m \times 1800 + C$
$135 = (m \times 3150) + C$

$m = \frac{1}{30}$
$C = 30$

$P = \frac{1}{30}W + 30$

(b) $P = \left(\frac{1}{30} \times 4500\right) + 30 = 180N$

(c) $V.R = \frac{l}{r}\left(\frac{T_1 \times T_3}{T_2 \times T_4}\right) = \frac{360}{80}\left(\frac{75}{20} \times \frac{90}{30}\right)$
$= 50.6$

$MA = \frac{W}{P} = 25$

$\eta = \frac{MA}{V.R} = 49.4\%$

(d) $\eta_{max} = \frac{1}{m \times V.R}$

Q/ In a single purchase crab winch, length of handle is 160mm and the gear ratio is 5. Find the vel ratio and $\eta_{m/c}$, if load of 1KN is lifted by an effort of 50N. Take dia of drum = 60mm.

$\eta = 5$

Simple Screw Jack.

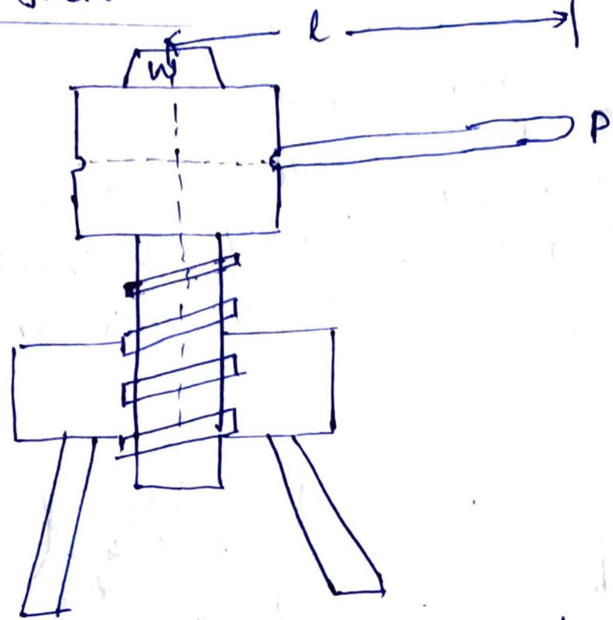


fig shows a simple screw jack, which is rotated by the application of an effort at the end of the lever, for lifting the load.

l = length of effort Arm

p = pitch of screw

W = load lifted

P = Effort applied to lift the load at the end of the lever.

dist moved by effort in 1 rev of screw $= 2\pi l$

" " load $= p$

$$MA = \frac{W}{P}$$

$$V.R = \frac{\text{dist moved by effort}}{\text{dist moved by Load}} = \frac{2\pi l}{p}$$

$$\eta = \frac{MA}{VR}$$

Q/ A screw jack has a thread of 10mm pitch. What effort applied at the end of a handle 400mm long will be required to lift a load of 2kN, if the η at this load is 45%..

Ans/

$VR = \frac{2\pi l}{p} = 251.3$

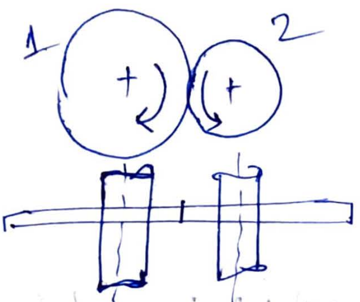
$MA = \frac{W}{P} = \frac{2000}{P}$

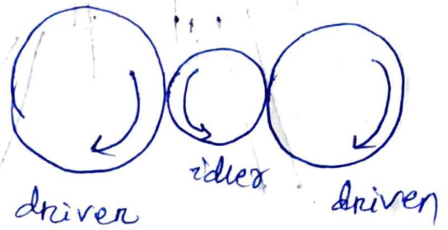
$\eta = \frac{MA}{VR} = \frac{2000/P}{251.3}$

$P = 17.7\ N$

Simple gear train

- 2 or more gears are made to mesh with each other to transmit power from one shaft to another such combination is called gear train.
- When distance betⁿ 2 shaft is small the 2 shaft 1 & 2 are made to mesh with each other.





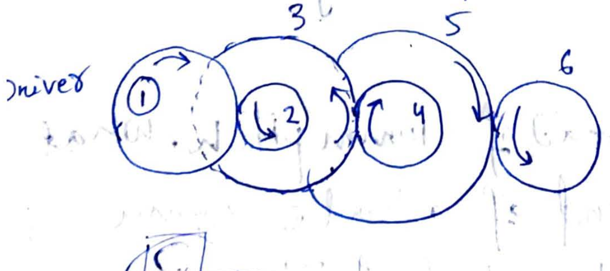
N_1 = speed of gear 1 (rpm)
N_2 = " " 2
T_1 = no. of teeth in gear 1
T_2 = " " 2

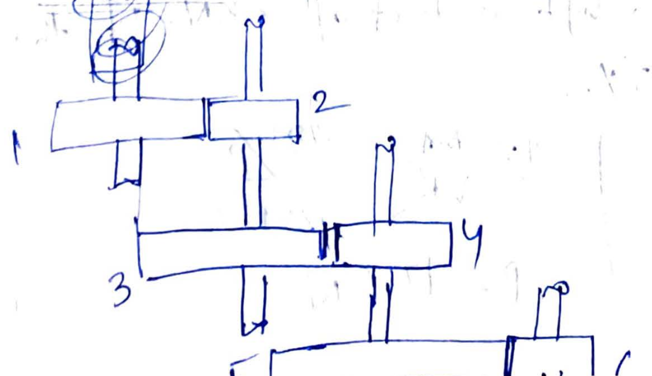
speed ratio $= \frac{N_1}{N_2} = \frac{T_2}{T_1} = \frac{\text{speed of Driver}}{\text{speed of driven}}$

Train value $= \frac{1}{\text{Speed ratio}} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$

Compound gear train.

when there are more than one gear on shaft it is called compound gear train.





N_1 = speed of driving gear 1
T_1 = no. of teeth on driving gear 1
$N_2, N_3, \ldots N_6$ = speed of respective gears in rpm
$T_2, T_3, \ldots T_6$ = no. of teeth on " "

gear 1 is mesh with gear 2.

So, speed ratio, $\frac{N_1}{N_2} = \frac{T_2}{T_1}$ —— (1)

for, 3, 4, $\frac{N_3}{N_4} = \frac{T_4}{T_3}$ —— (2)

for 5, 6, $\frac{N_5}{N_6} = \frac{T_6}{T_5}$ —— (3)

multiplying, 1, 2 & 3

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

Since, gear 2, 3 mounted on one shaft, so $N_2 = N_3$
gear 4, 5 " " " , $N_4 = N_5$

$$\boxed{\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}}$$ (speed ratio)

speed ratio = $\frac{\text{speed of 1st driver}}{\text{speed of last driven}}$

= $\frac{\text{product of no. of teeth on driven}}{\text{product of no. of teeth on driver}}$

$$\boxed{\text{Train value} = \frac{1}{\text{Speed ratio}}}$$