Impact OF Jet

The liquid comes out OF the pipe outlet through a nozzle. Nozzle has the outlet diameter very small compairing to pipe.

IF we apply continuity equation. $A_1V_1 = A_2V_2$

A_1 = Area OF pipe
V_1 = Velocity OF water in pipe

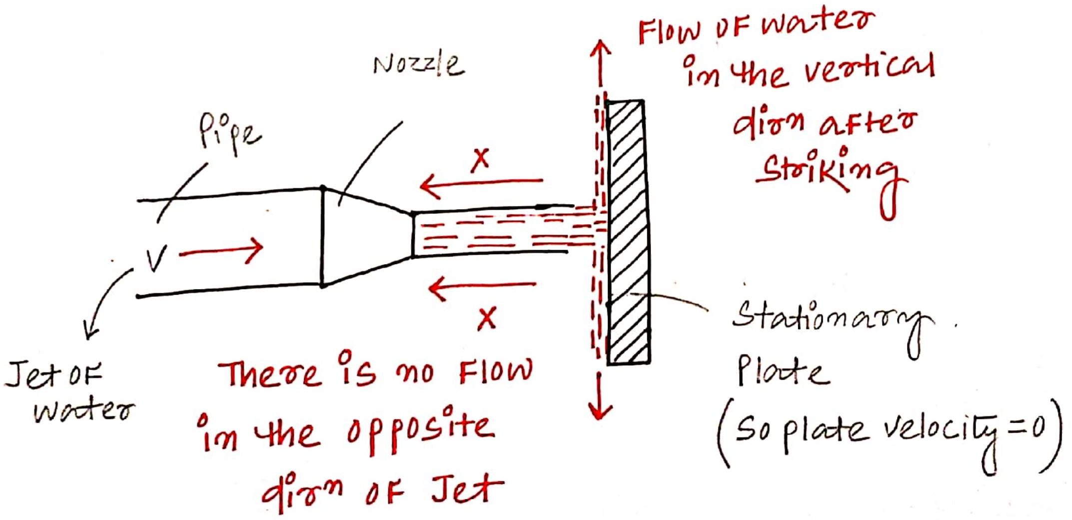
A_2 = Area OF ~~Inst~~ Nozzle outlet
V_2 = Velocity OF water at Nozzle outlet dia.

as $A_2 << A_1$ $V_2 >>> V_1$

Due to this water comes out OF Nozzle in a very high velocity forming jet. As the water comes with under pressure & velocity when it strikes on a Fixed or movable plate in front OF it it's velocity changes. As a result OF which there is a net change in momentum (M.V) which exerts a force on the plate due to which it may start moving or stay fixed still a net force is always applied on the plate

(a) Force exerted By the Jet on a Stationary vertical plate —

Consider a jet OF water coming out from Nozzle strikes a vertical plate which is stationary.



Let V_1 = velocity of Jet

d = diameter of Jet

a = area of x/s-c of Jet = $\frac{\pi}{4}d^2$

The Jet after striking the plate will move along the plate.

As the plate lies 90° to the dirn of Jet so flow occurs along the ~~Jet~~ plate.

There is no flow across the dirn of Jet after the strike.

$$F_x = \frac{d(mV)}{dt}$$ (Force = Rate of change in momentum in dirn of force. (Newton's 2nd Law))

$$F_x = \frac{M_1V_1 - M_2V_2}{\text{Time}}$$

$$= \frac{\text{Mass}}{\text{Time}}(V_1 - V_2) = \frac{m}{t}\left[\text{vel of Jet before striking} - \text{vel of Jet after striking}\right]$$

(mass remains const)

(continuity eqn)

$$= \dot{m}\,[V_1 - 0]$$

$$= \rho . a\, V_1 . V_1 = \rho a V_1^2$$

$[\dot{m} = \rho . a . V]$

$Kg/s \quad (Kg/m^3 \times m^2 \times \frac{m}{s})$

$$\boxed{F_x = \rho a V^2.}$$

Here as the plate is stationary

Force x displacement = 0

displacement $\to 0$

So Jet can not produce any workdone.

$W = F \times S = 0$

N.B

here we are calculating Force exerted by Jet on the plate

So $\frac{d(mv)}{dt} = m_1V_1 - m_2V_2$

If Force exerted on Jet by plate asked

$$\frac{d(mv)}{dt} = m_2V_2 - m_1V_1$$

(2) Force on Flat vertical plate moving in the dirn of Jet —

Here when the Jet strikes the plate with a velocity 'V' plate also moves with a velocity 'u' along the dirn of Jet.

So the relative velocity of the Jet with respect to plate $= V - u$

$\xrightarrow{V}$ $\xrightarrow{u}$

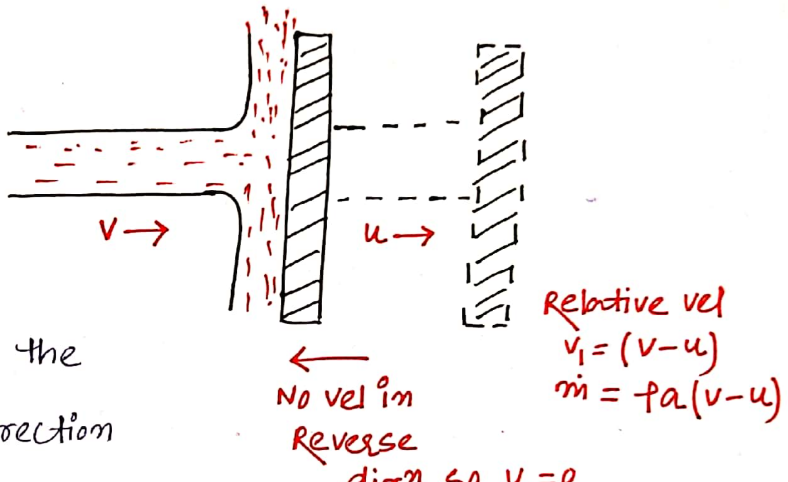
Mass of water striking the plate per sec

$= \rho \times$ Area of Jet $\times$ Velocity with which Jet strikes

$\dot{m} = \rho \times a \times (V-u)$

V_1 = Initial vel. with which Jet strikes $= V - u$

V_2 = Final vel in the opposite dirn of Jet $= 0$



Force Exerted by the Jet on the moving plate in the Jet direction

$$= F_x = \frac{d(mV)}{dt}$$

$$= \frac{m_1 V_1 - m_2 V_2}{t} = \frac{m}{t}(V_1 - V_2) = \dot{m}(V_1 - V_2)$$

$$= \rho . a(V-u)\left[(V-u) - 0\right] = \rho a (V-u)^2$$

$$\boxed{F_x = \rho a (V-u)^2}$$

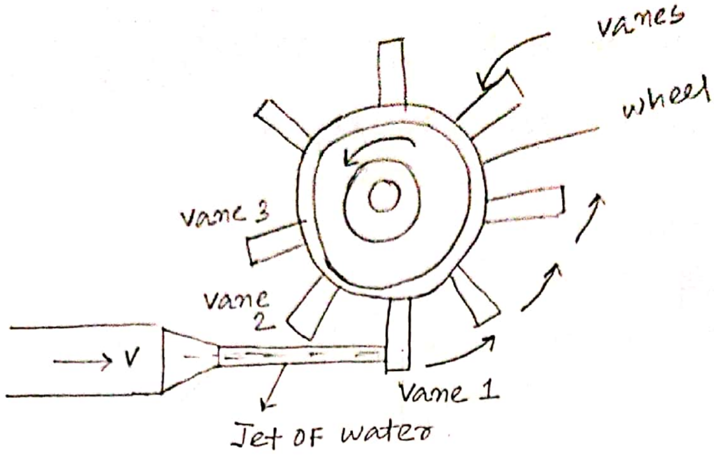
workdone per sec $=$ Force $\times \frac{\text{displacement}}{\text{sec}}$

$=$ Force $\times$ Velocity

$= F \times$ plate velocity

$$\boxed{\frac{W}{S} = \rho a (V-u)^2 \times u}$$

(3) Force exerted on series of vanes —



when the Jet strikes on the plate and due to the force exerted by the Jet on the plate the wheel starts Rotating.

As soon as Vane 1 starts to rotate immediately vane 2 takes its place there vane 3 and so on.

The wheel moves in a constant speed 'u'.

here the mass of water coming out from Nozzle per second always remains in contact with vanes.

So mass of water per second striking the series of plate

$$\dot{m} = \rho a V$$

Relative velocity of Jet w.r.t to plate $(V_1) = V - u$ & $V_2 = 0$

Force exerted by jet in dirn of motion of plate

$$F_x = \frac{d\,mv}{dt} = \frac{m_1 V_1 - m_2 V_2}{t} = \frac{m}{t}(V_1 - V_2) = \dot{m}(V_1 - V_2)$$

$$= \rho a V[(V-u) - 0]$$

$$= \rho a V (V-u)$$

$$\boxed{F_x = \rho a V (V-u)}$$

$$\frac{\text{workdone}}{\text{sec}} = F_x \times u = \rho a V (V-u) \cdot u$$

Kinetic energy of Jet per sec $= \frac{1}{2} m v^2$

$= \frac{1}{2} (\rho a v) v^2 = \frac{1}{2} \rho a v^3$

due to this Kinetic energy of Jet we are getting workdone.

As more KE → In more speed it will impinge on vane
↓
more Force exerted
↓
more workdone.

So $\eta = \frac{\text{output}}{\text{Input}}$

$= \frac{\text{workdone per sec}}{\text{K.E per sec}} = \frac{\rho a v (v-u) \cdot u}{\frac{1}{2} \rho a v^3} = \frac{2u(v-u)}{v^2}$

Conditions for Maximum efficiency —

efficiency will be Maximum when

$\frac{d\eta}{du} = 0$

$\frac{d\left(\frac{2u(v-u)}{v^2}\right)}{du} = 0 \Rightarrow \frac{d}{du}\left(\frac{2uv - 2u^2}{v^2}\right) = 0 \Rightarrow \frac{2v - 2(2u)}{v^2} = 0 \Rightarrow 2v - 4u = 0$

$\Rightarrow 2v = 4u \Rightarrow v = 2u \quad \boxed{u = \frac{v}{2}}$

η will be max^m when plate velocity will be half of Jet velocity.

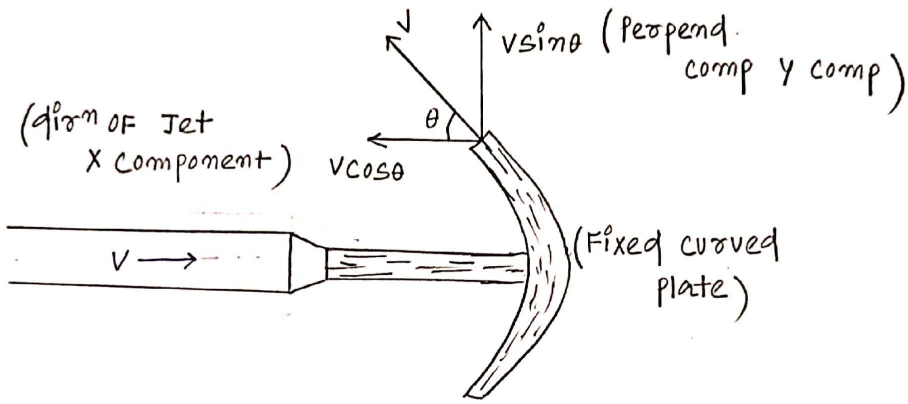
$\eta_{max} = \frac{2u(v-u)}{v^2} \quad \left[\text{Put } u = \frac{v}{2},\ v = 2u\right]$

$= \frac{2u(2u-u)}{(2u)^2} = \frac{1}{2} = 0.5$ or 50%

Force exerted on stationary curved plate —

Let Jet of water strikes a curved plate at centre. The Jet after striking the plate comes with same vel. in the tangential dirn of curved plate.

The velocity at outlet can be resolved into two components one in dirn of Jet and other perpendicular to it.



Component of vel in the dirn of Jet $= -V\cos\theta$

-ve sign as dirn of vel is in opposite dirn than inlet velocity.

Component of velocity in perpendicular dirn $= V\sin\theta$

Force exerted by the Jet in the dirn of Jet =

$$F_x = \dot{m}(V_1 - V_2)$$

$$= \dot{m}(V - -V\cos\theta) = \rho a V (V + V\cos\theta)$$

$$= \rho a V . V (1 + \cos\theta) = \rho a V^2 (1 + \cos\theta)$$

F_y = Force exerted on perpendicular dirn

$$= \dot{m}(V_1 - V_2)$$

$$= \rho a V (0 - V\sin\theta) = -\rho a V^2 \sin\theta$$

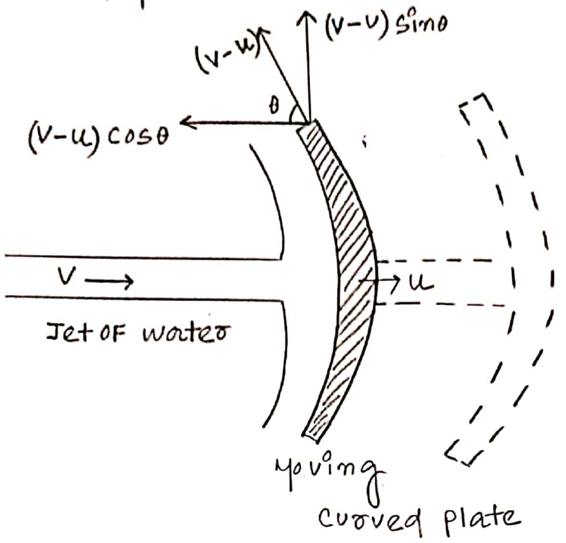
-ve sign means Force is acting in downward dirn.

- **Force on curved plate when plate is moving in dirⁿ of Jet —**

V = absolute vel of Jet

u = vel of plate in the dirⁿ of Jet.

$V-u$ = Relative vel of Jet of water with which it strikes the curved plate.



$F_x = \dot m \times$ component of vel. in the dirⁿ of Jet (Initial − Final)

$F_y = \dot m \times$ component of vel in the perpendicular dirⁿ of Jet (Initial − Final)

$\dot m = \rho \times a \times$ vel. at which Jet strikes the plate

$= \rho a (V-u)$

$$F_x = \dot m \times (V_1 - V_2) = \dot m\left[(V-u) - \left(-(V-u)\cos\theta\right)\right]$$

$$= \dot m\left[(V-u) - \left[-(V-u)\cos\theta\right]\right]$$

$$= \rho a (V-u)\left[(V-u) + (V-u)\cos\theta\right]$$

$$= \rho a (V-u)^2 \left[1+\cos\theta\right]$$

$$F_y = \dot m\left[0 - (V-u)\sin\theta\right] = \rho a (V-u)\left[-(V-u)\sin\theta\right] = -\rho a (V-u)^2 \sin\theta$$

Workdone by the Jet on the plate per second —

$W = F_x \times$ distance travelled per sec in dirⁿ of Jet

$$= F_x \times u = \rho a (V-u)^2 (1+\cos\theta) \cdot u$$

$$W = \rho a (V-u)^2 \cdot u \left[1+\cos\theta\right]$$

(3) Jet strikes the curved plate at one end tangentially when plate is symmetrical —

V = Velocity of Jet of water

θ = angle made by Jet with x axis at inlet tip of curved plate.

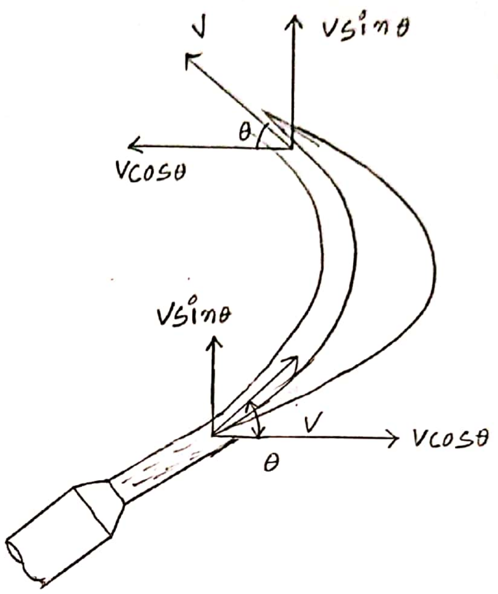
$$F_x = \dot{m}(V_{1x} - V_{2x})$$

$$= \dot{m}[V\cos\theta - (-V\cos\theta)]$$

$$= \rho a v (V\cos\theta + V\cos\theta)$$

$$= \rho a v^2 (\cos\theta + \cos\theta)$$

$$= 2\rho a v^2 \cos\theta$$



$$F_y = \dot{m}(V_{1y} - V_{2y})$$

$$= \rho a v (V\sin\theta - V\sin\theta)$$

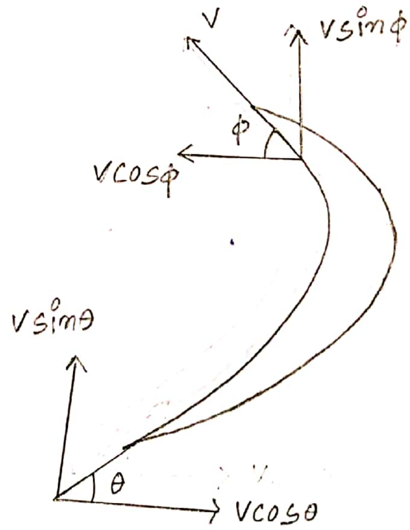
$$= 0$$

same dir^n upward.

(4) when plate is unsymmetrical —

θ = angle made by tangent at Inlet tip with x axis

ϕ = angle made at outlet tip with x axis.



$$F_x = \dot{m}(V_1 - V_2)$$

$$= \rho a v [V\cos\theta - (-V\cos\phi)]$$

$$= \rho a v^2 (\cos\theta + \cos\phi)$$

$$F_y = \rho a v (V\sin\theta - V\sin\phi)$$

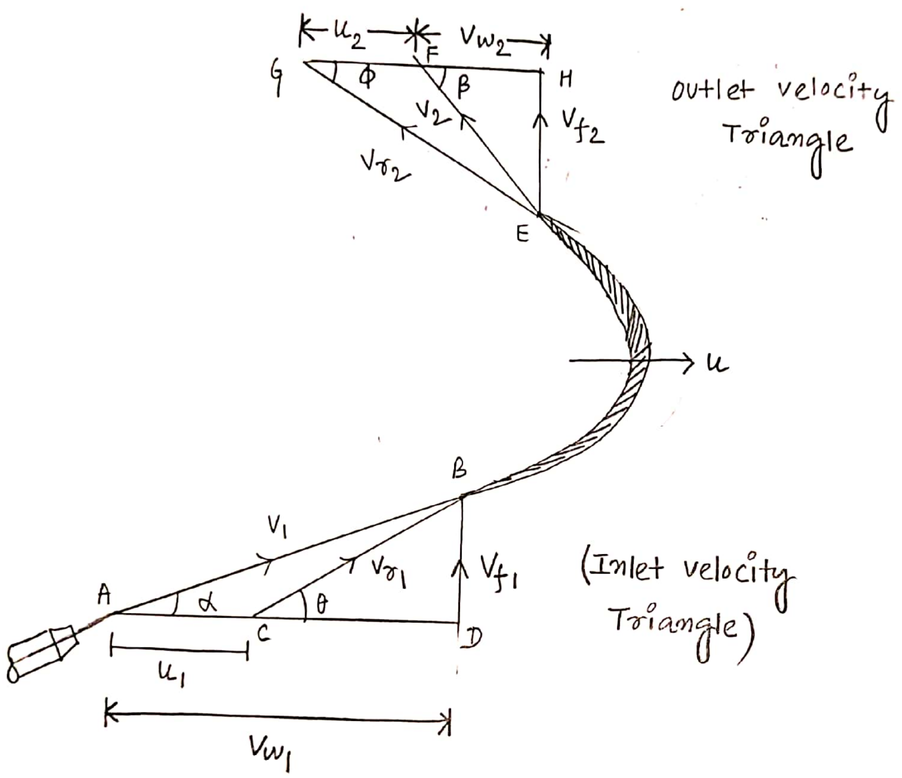
$$= \rho a v^2 (\sin\theta - \sin\phi)$$

V. Imp.

⑤ Force Exerted by a Jet of water on an unsymmetrical moving curved plate when Jet strikes tangentially at one Tip.

* [Introduction of velocity Triangle]

In this case as plate is moving the velocity with which Jet of water strikes is equal to the relative vel of Jet wrt plate.



V_1 = Velocity of Jet at Inlet.

u_1 = Velocity of plate (vane) at Inlet

V_{r_1} = Relative vel. of Jet and plate at Inlet

α = angle between dirn of Jet and direction of motion of plate

(Guide Blade angle)

θ = angle betn Relative vel. of Jet and plate with dirn of motion of plate

(vane angle at inlet)

V_{w_1} = The component of vel of Jet V_1 in the dirn of motion.
Known as velocity of of whirl at inlet

V_{f_1} = Component of vel of Jet V_1 in perpendicular dirn.
Known as vel. of Flow at outlet.

V_2 = velocity of Jet leaving the vane at outlet.

u_2 = Velocity of vane at outlet.

V_{r_2} = Relative vel. of Jet w.r.t vane at outlet.

β = angle made by velocity V_2 with dirn of motion at outlet.

ϕ = angle made by relative vel. V_{r_2} " " "
Known as vane angle at outlet.

V_{w_2} = Velocity of whirl at outlet.

V_{f_2} = Velocity of Flow at outlet.

If the vane is smooth and have equal velocity at inlet and outlet then —

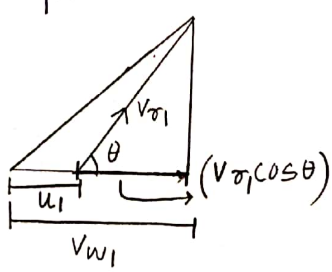
$$u_1 = u_2 = u = \frac{\pi D N}{60}$$

$$V_{r_1} = V_{r_2}$$

Mass of water striking vane per sec $= \rho a V_{r_1}$

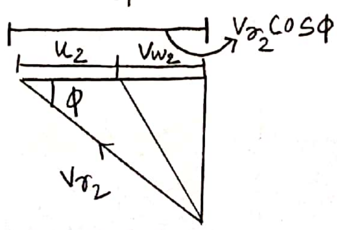
Component of Relative velocity in the dirn of motion

$$= V_{r_1}\cos\theta = V_{w_1} - u_1$$



Component of Relative velocity at outlet in dirn of motion

$$= -V_{r_2}\cos\phi = -(u_2 + V_{w_2})$$



Force Exerted by the Jet in the direction of motion

F_x = mass of water striking per sec

× [Initial vel with which Jet strikes in the dirn of motion − Final vel. in the dirn of motion]

$$F_x = \dot{m}\,(V_{1x} - V_{2x})$$

$$= \rho a V_{r_1}\left[V_{r_1}\cos\theta - (-V_{r_2}\cos\phi)\right]$$

$$= \rho a V_{r_1}\left[(V_{w_1} - u_1) - \{-(u_2 + V_{w_2})\}\right]$$

$$= \rho a V_{r_1}\,(V_{w_1} - u_1 + u_2 + V_{w_2}) = \rho a V_{r_1}(V_{w_1} + V_{w_2}) \quad [\text{as } u_1 = u_2]$$

If $\beta < 90^\circ$ $\quad F_x = \rho a V_{r_1}(V_{w_1} + V_{w_2})$

$\beta = 90^\circ$ $\quad F_x = \rho a V_{r_1} V_{w_1}$ $\quad (V_{w_2} = 0$ if $\beta = 90^\circ)$

$\beta > 90^\circ$ $\quad F_x = \rho a V_{r_1}(V_{w_1} - V_{w_2})$

In general $\quad F_x = \rho a V_{r_1}(V_{w_1} \pm V_{w_2})$

Workdone per sec on the vane by the Jet

= Force × distance per sec in the dirn of Force

$= F_x \times u$

$$W/sec = \rho a V_{r_1}(V_{w_1} \pm V_{w_2}) \times u$$

Workdone per sec per unit weight ~~of~~ of fluid striking

$$\text{per sec} = \frac{\rho a V_{r_1}(V_{w_1} \pm V_{w_2}) \cdot u}{mg} = \frac{\rho a V_{r_1}(V_{w_1} \pm V_{w_2}) \cdot u}{\rho a V_{r_1} \cdot g}$$

$$= \frac{1}{g}(V_{w_1} \pm V_{w_2}) \cdot u$$

$$W.D/sec \text{ per unit mass of Fluid striking} = \frac{\rho a V_{r_1}(V_{w_1} \pm V_{w_2}) \cdot u}{\rho a V_{r_1}}$$

$$= (V_{w_1} \pm V_{w_2}) \cdot u$$

Efficiency of Jet —

$$\eta = \frac{\text{output}}{\text{Input}} = \frac{\text{workdone per sec on the vane}}{\text{Initial KE per sec of Jet}}$$

$$= \frac{\rho a V_{r_1} (V_{w_1} \pm V_{w_2}) \cdot u}{\frac{1}{2} m V_1^2}$$

$$= \frac{\rho a V_{r_1} (V_{w_1} \pm V_{w_2}) \cdot u}{\frac{1}{2} (\rho a V_1) \cdot V_1^2}$$

Question Find the force exerted by a jet of water of diameter 75 mm on a stationary flat plate, when the jet strikes the plate normally with velocity of 20 m/s.

Ans. Given :

Diameter of jet, $d = 75 \text{ mm} = 0.075 \text{ m}$

$\therefore$ Area, $a \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.075)^2 = 0.004417 \text{ m}^2$

Velocity of jet. $V = 20$ m/s.

The force exerted by the jet of water on a stationary vertical plate is given by equation

$$F = \rho a V^2 \text{ where } \rho = 1000 \text{ kg/m}^3$$

$$\therefore F = 1000 \times 0.004417 \times 20^2 \text{N} = 1766.8 \text{ N. Ans.}$$

Question Water is flowing through a pipe at the end of which a nozzle is fitted. The diameter of the nozzle is 100 mm and the head of water at the center nozzle is 100m. Find the force exerted by the jet of water on a fixed vertical plate. The co-efficient of velocity is given as 0.95.

Ans. Given :

Diameter of nozzle, $d = 100$ mm $= 0.1$ m

Head of water. $H = 100$ m

Co-efficient of velocity, $C_1 = 0.95$

Area of nozzle, $a = \frac{\pi}{4}(.1)^2 = .007854 \text{ m}^2$

Theoretical velocity of jet of water is given is as

$$v_{th} = \sqrt{2gH} = \sqrt{2X9.81 \times 100} = 44.294 \text{ m/s}$$

But $$c_v = \frac{\text{Actual velocity}}{\text{Theoretical velocity}}$$

$\therefore$ Actual velocity of jet of water. $V = C_v X v_{th} = 0.95 \times 44.294$

$= 42.08$ m/s.

Force on a fixed vertical plate is given by equation as

$F = paV^2 = 1000 \times .007854 \times 42.08^2$ ($\because$ In S.I. units p for water $= 1000$ kg/m^2)

$= 13907.2$ N $= 13.9$ kN. Ans.

Question A jet of water of diameter 75 mm moving with a velocity of 25m/s strikes a fixed plate in such a way that the angle between the jet and plate is 60°. Find the force exerted by the jet on the plate (i) in the direction normal to the plate and (ii) in the direction of the jet.

Solution. Given :

Diameter of jet, $d = 75$ mm $= 0.075$ m

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.075)^2 = 0.004417$ m^2

Velocity of jet. $V = 25$ m/s.

Angle between jet and plate $\theta = 60°$

The force exerted by the jet of water in the direction normal to the plate is given by equation

$$F_n = paV^2 \sin\theta$$

$$= 1000 \times .004417 \times 25^2 \times \sin 60° = 2390.7 \text{ N. Ans.}$$

Question A jet of water of diameter 50 mm strikes a fixed plate in such a way that the angle between the plate and the jet is 30;; the force exerted in the direction of the jet is 1471.5 N. Determine the rate of flow of water.

Solution:- Given :

Diameter of jet, $d = 50\ mm = 0.05\ m$

Area $a = \frac{x}{4}(.05)^2 = .00196\ m^2$

Angle, $\theta = 30^0$

Force in the dürection of jet, $F_x = 1471.5\ N$

Force in the direction of jet is given by eduction (17.3) as $F_1 = pav^2 \sin^2\theta$

As the force is given in Newton, the value of p should be taken equal to 1000 kg/m^3

$$1471.5 = 1000 \times .001963 \times V^2 \times \sin^2 30^0 = .05\ V^2$$

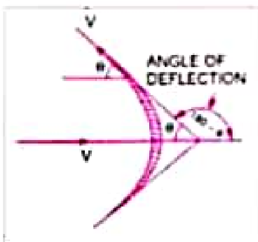
$$V^2 = \frac{150}{.05} = 3000.0$$

$$V = 54.77\ m/s$$

Discharge, $Q = \text{Area} \times \text{Velocity}$

$$= .001963 \times 54.77 = 0.01075\ m^3/s = 107.5\ \text{liters/s. Ans}$$

Question A jet of water of diameter 50mm moving with a velocity of 40m/s, strikes a curved jet if the jet is through an angle of 120^0 at the outer of the curved plate



Solution:- Given

Diameter of the jet, $d = 50\ mm = 0.05m$

Area, $a = \frac{\pi}{4}(.05)^2 = 0.001963\ m^2$

Velocity of jet, $V = 40$ m/s

Angle of defiction $= 120^0$

From equation (17.6 A) the angle of deflection $= 180^0 - \theta$

$$180^0 - \theta = 120 \text{ or } \theta = 180^0 - 120^0 = 60^0$$

Force excrted by the jet on the curved plate in the direction of the jet is given by equation

$$F_x = pav^2 (1 + \cos\theta)$$

$$= 1000 \times .001963 \times 40^2 \times (1 + \cos 60^0) = 4711.15 \text{ N. Ans}$$

Question A jet of water of 30 mm diameter strikers a hinged square plate at its centre with a velocity of 20 m/s. The plate is deflected through an angle of 20^0 Find the weight of the plate.

If the plate is not allowed to swing what will be the force required at the lower wdge of the plate to keep the plate in vertical position.

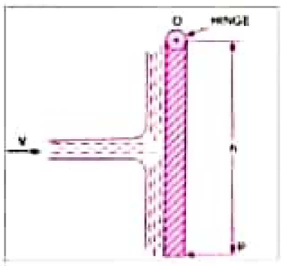
Solution:- Given :

Diameter of the jet, $d = 30$ mm $= 3$ cm $= 0.03$ m

Area, $a = \frac{x}{4} d^2 = \frac{x}{4} (.03)2 = .0007068\ m^2$

Velocity of jet, $V = 20$ m/s

Angle of swing, $\theta = 20^0$



Using eqation for angle of swing

$$\text{Sin } \theta = \frac{pav2}{w}$$

Or $$\sin 20^0 = 1000 \times \frac{.0007068 \times 20}{w} = 282.72$$

$$W = \frac{282.72}{\sin 20} = 826.6 \text{ N}$$

If the plate is not allowed to swing a force P will applied at the lower edge of the plate as shown in Fig The weight of the plate is acting vertically downward through the C.G of the plate,

Let F = force exerted by jet of water

h= height of plate

= Distance of P from the hinge.

The jet strikes at the center of the plate and hence distance of the centre of the jet from hinge = $\frac{h}{2}$

Taking moments about the hinge O. $P \times h = F \times \frac{h}{2}$

$$P = \frac{F \times h}{2 \times h} = \frac{F}{2} = \frac{pav2}{2}$$

$$= 1000 \times \frac{.0007068 \times 20}{2} = 14136 \text{ Ans}$$