GOVT. POLYTECHNIC, ANGUL

LECTURE NOTES WITH QUESTION BANK

(Mechanical Department)

COURSE: -STRENGTH OF MATERIAL

(ASSESMENT YEAR-2021-22)

CODE- Th-2

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LOAD :-

Load is defined as a force tends to produce an external effect of a body and produce deformation or displacement from one position to the other position.

COMMON COLD :-

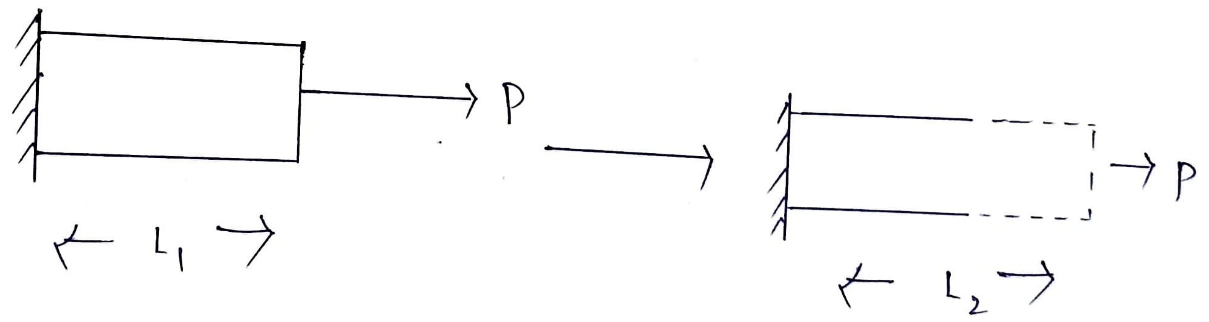
common load in engineering application are tension and compression.

i.e - load is divided in two types.

(i) Tensile load (Direct pull)

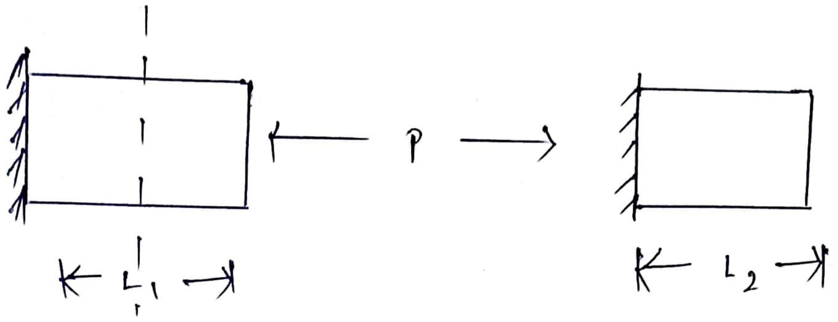
(ii) compressive load (Direct push)

(i) Tensile load (Direct pull) :-



$L_2 > L_1$, Because of application of tensile load length is increase.

(ii) compressive load :-



$L_2 < L_1$, Because of application of compressive load with is increases.

TYPES OF LOAD :-

There are a no of different ways in which load can be applied to a member the tipically loading are:

(i) Dead / static load

(ii) Live / Dynamic load

(iii) Impact / shuck load

(iv) cyclic / repeated load

(i) Dead / static load ÷

There is no fluctuation/ displacement because of the application of load. These load are always constant.

E.g ÷ weight of beam, weight of floor etc.

(ii) Live / Dynamic load ÷

These Dynamic load are otherwise known as movable load that means this load is unstable and donot remain constant.

E.g ÷ the load exerted by people, furniture, equipments etc.

(iii) Impact / shuck load ÷

It is the strain load that is apply on a stationary or movable object. It will act for a fraction of second.

Eg ÷ An object within very high speed hit another object.

(iv) cyclic / repeated load ÷

The load which repeats after each and every cycle is known as cyclic load or repeated load.

E.g ÷ These loads are repeated loading on a beam or on a structure.

LOAD

- W.r.t time
  - Static
    - Dead load
    - Gradially load
  - Dynamic
    - Impact load applied
    - cyclic load / fatigue load
- W.r.t cross section
  - Normal Load
    - Axial load
    - Eccentric axial load
    - Tensile/compressive Load
  - Shear/Tangential load
    - Transverse load
    - Eccentric Transverse load
- W.r.t distribution of Load
  - Concentreted
  - Distributed
    - UDL
    - UVL
    - UDM

UDL → Uniform Distribute Load

UVL → Uniform variable Load

UDM → Uniform Distributed motion

STRESS :-

when an external force is applied on a body some internal resisting force is set up with in the material this internal resisting force per unit area of cross section is known as stress .

$$\sigma = \frac{P}{A}$$

unit N/m^2 & dyn/cm^2

$$1\ N/m^2 = 1\ kg \cdot \frac{m}{sec^2} \times \frac{1}{m^2}$$

$$= \frac{10^3\ gm \cdot 100\ cm}{sec^2 \times (100)^2\ cm^2}$$

$$= \frac{10^3 \times 10^2}{10^4}\ \frac{gm}{cm \cdot sec^2}$$

$$= \frac{10^5}{10^4}\ \frac{gm}{cm \cdot sec^2}$$

$$= 10\ \frac{gm}{cm \cdot sec^2}$$

$$1 \cdot Nm^2 = 10\ \frac{dyn}{cm^2}$$

$$1\,N = 1\,kg \cdot \frac{m}{sec^2}$$

$$= 10^3 \frac{gm \cdot 100\,cm}{sec^2}$$

$$= 10^5\ gm \cdot cm/sec^2$$

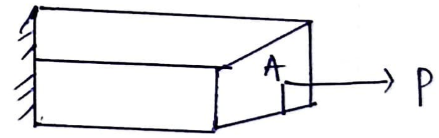
$$\boxed{1N = 10^5\ dyne}$$

Stress is two types :-

(i) Normal / Axial stress :-

When the load is applied ⊥ʳ to the area of cross section then it is called Normal stress.

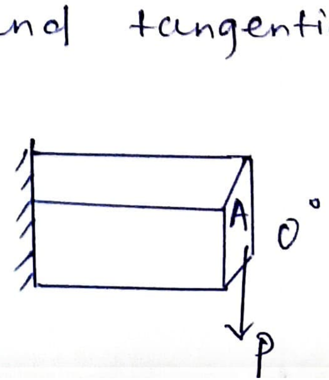
$$\sigma_N = \frac{P}{A}$$



(ii) Shear / Tangential stress :-

When the load is applied ∥ʳ on the area of cross section then the stress generated due to this is known as shear and tangential stress.

$$\tau = \frac{P}{A}$$



STRAIN :-

strain is defined as the ratio between the change in dimension to the original dimension.

$$E = \frac{\text{change in dimension}}{\text{Original dimension}}$$

3 types of strain :-

(i) Longitudinal strain (ϵ_l)

(ii) Lateral strain (ϵ_b)

(iii) volumetric strain (ϵ_v)

(i) Longitudinal strain :-

Longitudinal strain is defined as the ratio between change in length to the Original length.

$$\epsilon_L = \frac{\Delta L}{L}$$

(ii) Lateral strain :-

Lateral strain is defined as the ratio between change in width to the original width.

$$\epsilon_b = \frac{\Delta b}{b}$$

(iii) volumetric strain :-

volumetric strain is defined as the ratio between change in volume to the Original volume.

$$\varepsilon_v = \frac{\Delta V}{V}$$

HOOKE'S LAW :-

The Robert Hooke and english Mathmetician perform many experiments in order to find the relation between stress and strain. He found that stress is directly propertional to the strain and is the valid upto elastic limit only.

Statement :- When a body is loaded within the elastic limit stress is directly proportional to the strain.

So, Mathmatically

$$\sigma \propto \epsilon$$

$$\sigma = E\epsilon$$

Where, 'E' is Modulus of elastcity or Elastic constant.

'σ' is Normal stress.

'ϵ' is longitudinal strain.

ELASTIC CONSTANT :-

Elastic constant are of '3' types.

(a) Modulus of elasticity :-

$$E = \frac{\text{Normal Stress}}{\text{Longitudinal strain}}$$

$$E = \frac{\sigma_N}{\varepsilon_L}$$

(b) Modulus of Rigidity :-

$$G = \frac{\text{shear stress}}{\text{shear strain}} = \frac{\tau}{\phi}$$

(c) Bulk's Modulus :-

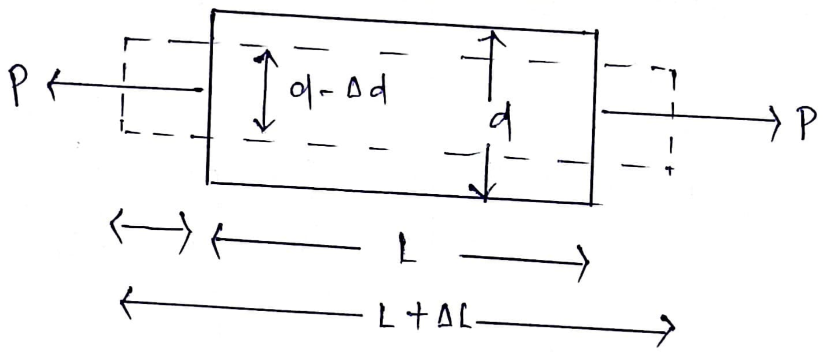
$$K = \frac{\text{Normal stress}}{\text{Volumetric strain}}$$

$$\boxed{K = \frac{\sigma}{\epsilon_v}}$$

POISSON'S RATIO (V):

Poisson's ratio is defined as the ratio between lateral strain to the longitudinal strain.

$$V = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \left(\frac{\text{Change in dia}}{\text{Original dia}}\right) \Big/ \left(\frac{\text{change in length}}{\text{Original length}}\right)$$



Change in dia = Final dia − initial dia

$$= (d - \Delta d) - d = -\Delta d$$

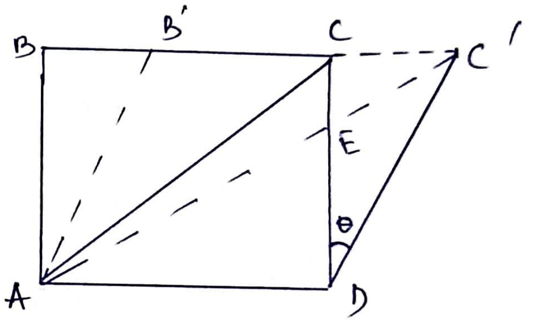
$$\text{change in length} = (L + \Delta L) - L = \Delta L$$

$$V = \frac{\left(\frac{-\Delta d}{d}\right)}{\left(\frac{\Delta L}{L}\right)}$$

$$\boxed{V = -\left(\frac{\Delta d}{d}\right) \times \left(\frac{L}{\Delta L}\right)}$$

Derivation of relationship between E & G:-

Let us conside a square block ABCD as shown in the figure. If we applied some amount of food then the point c. the diagonal if we draw CE $\perp^{r}$ AC' so in the increasing in length of the diagonal 'AC' will 'EC' Because 'EC' is the change in length of the diagonal AC.

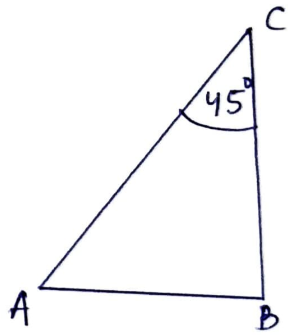


In Δ CC'E

$$\cos 45^\circ = \frac{EC'}{CC'}$$

$$\Rightarrow EC' = CC' \cos 45^\circ$$

$$\Rightarrow \cos 45^\circ = \frac{EC'}{CC'}$$



strain along diagonal

$$AC = \frac{EC'}{AC} = \frac{\Delta L}{L}$$

$$\cos 45^\circ = \frac{DC}{AC}$$

$$= \frac{CC' \cos 45^\circ}{\left(\frac{DC}{\cos 45^\circ}\right)}$$

$$= \left(\frac{CC'}{DC}\right) \cos^2 45^\circ$$

$$= \left(\frac{cc'}{DC} \right) \times \frac{1}{2}$$

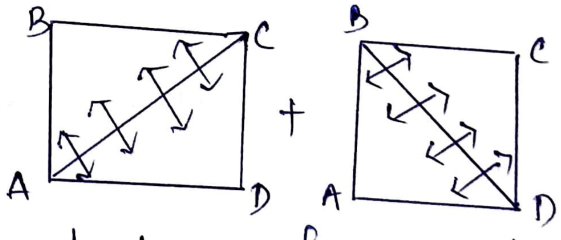
$$= \phi / 2$$

$$\Rightarrow \boxed{\epsilon = \frac{\phi}{2}} \text{ ——— (i)}$$

Strain along Diagonal AC = strain produce by the tensile F along AC + the strain produce by the compressive force along bd.

$$AC = \frac{\tau}{E} + v \frac{Z}{E}$$

$$\Rightarrow \boxed{\epsilon = \frac{Z}{E}(1+v)}$$



From defination of modulus of Rigidity :-

We know that

$$\boxed{G = \frac{Z}{\phi} \Rightarrow \phi = \frac{Z}{G}}$$

We can replace it in eq$^{(n)}$ (i) so,

$$\boxed{\epsilon = \frac{\phi}{2} = \frac{\tau}{2G}} \text{ ——— (3)}$$

From eqn (2) and (3)

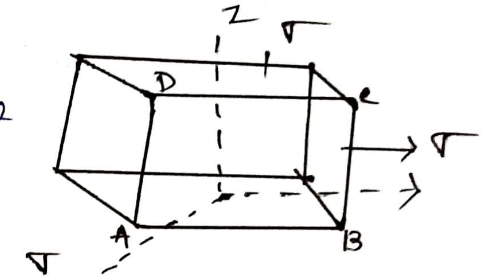
we have,

$$\frac{t}{2G} = \frac{\tau}{E}(1+\nu)$$

$$\Rightarrow \boxed{E = 2G(1+\nu)}$$

Relation between young's modulus (E) & Bulk modulus (K) :-

Let us conside a cube of side length 'l' as shown in the figure.



There are force volume of the cube.

V = l3

Differentation both side w.r.t 'l'.

$$\frac{dv}{dl} = \frac{l}{dl}(l^3)$$

$$= 3l^2$$

Now,
$$\frac{\Delta V}{\Delta l} = 3l^2$$

$$\Rightarrow \Delta V = 3l^2 \cdot \Delta l$$

$$\Rightarrow \frac{\Delta V}{V} = 3l^2 \frac{\Delta l}{V}$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{3l^2 \Delta l}{L^3}$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{3\Delta l}{l}$$

$$\Rightarrow \boxed{\epsilon_v = 3\epsilon_L}$$

Strain of DC due to σ along x-axis $= \frac{\sigma}{E}$

Strain of DC due to σ along y-axis $= -\sigma/mE$

Strain of DC due to σ along z-axis $= \frac{-\sigma}{mE}$

Total strain of DC due to $\sigma = \frac{\sigma}{E} - \frac{\sigma}{mE} - \frac{\sigma}{mE}$

$$\Rightarrow \frac{\sigma}{E}\left(1 - \frac{2}{m}\right) = \frac{\sigma}{E}(1-2v)$$

$$K = \frac{\sigma}{\epsilon_v}$$

$$\Rightarrow K = \frac{\sigma}{3\epsilon_L}$$

$$= \frac{\cancel{\sigma}}{\frac{3\cancel{\sigma}}{E}(1-2v)}$$ ——— from $Eq^{(n)}$ (i)

$$K = \frac{1}{\frac{3}{E}(1-2v)}$$

$$\Rightarrow \boxed{E = 3K(1-2v)}$$

where,

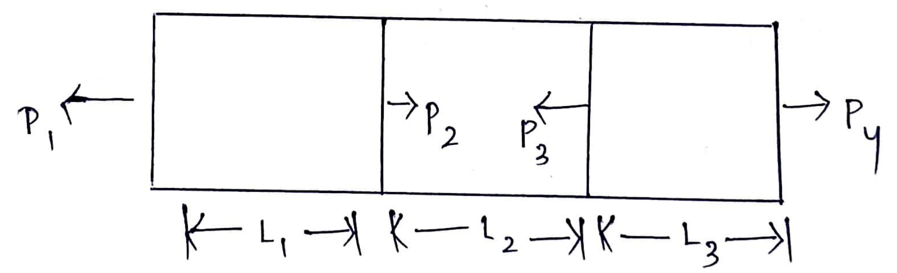
'E' is Modulus of elasticity.

'K' is Bulk modulus.

'ν' is poision ratio.

PRINCIPLE OF SUPERPOSITION :-

When a bar subjected to load at a different sections. it is subjected to deformation at indivisual section. Hence the totall deformation of the bar is equal to the sum of deformation of the indivisual section. such a principl of finally the total deformation is known as principle of superposition



According to the Hooke's law

$$\Delta L = \Delta L_1 + \Delta L_2 + \Delta L_3$$

$$\sigma \propto \epsilon$$

$$\Rightarrow \sigma = E\epsilon$$

$$\Rightarrow \epsilon = \frac{\sigma}{E}$$

$$\Rightarrow \frac{\Delta L}{L} = \frac{\sigma}{E}$$

$$= \left(\frac{P}{A}\right)\frac{L}{E}$$

$$\Rightarrow \boxed{\Delta L = \frac{PL}{AE}}$$

THERMAL STRESS :-

When there is change in temprerature of the material is subjected to same type of stress known as thermal stress Hence thermal stores depend on the change on temprerature the amount of thermal expansion depends on the co-efficient of thermal expansion. (α).

$$\boxed{\sigma_T = \alpha T E}$$

T → change in temprerature

E → Young modulus of elasticity

α → Co-efficient of linear expansion.

$\alpha_{steel} = 12 \times 10^{-6}/°C$ $\qquad$ $\alpha_{Al} = 23 \times 10^{-6}/°C$

$\alpha_{Cu} = 16 \times 10^{-6}/°C$

$\alpha_{brass} = 17 \times 10^{-6}/°C$

$\alpha_{gun\ metal} = 19 \times 10^{-6}/°C$

TEMPRATURE STRESS AND STRAIN :-

→ When the tempreature of a body is raised or lowered, there will be corresponding increase or decrease in its dimension.

→ If this change in dimensions is prevented by external forces, the body develops stress in it. These stress are known as tempreature stress. or thermal stress.

→ The corresponding strain is called tempreature strain or thermal strain.

CO-EFFICIENT OF LINEAR EXPANSION :-

→ The change in length of a body per unit change in tempreature per unit length is termed as the co-efficient of linear expansion.

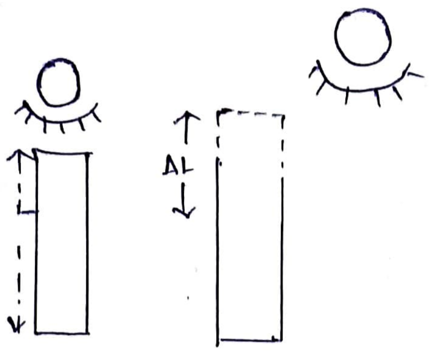
→ It is denoted by 'α' and expressed as $^{\circ}C, ^{\circ}F, ^{\circ}K$

TEMPERATURE STRESS :-

→ When a material undergoes natural exponsion or contraction as a result of rise or drop in tempreature, there is no strain and hence there will not be any stress in the material.

→ It is stress induced in a body due to prevention of deformation caused by change in the tempreature of the body.

→ The nature of tempreature stress induced depends on the nature of deformation prevented.



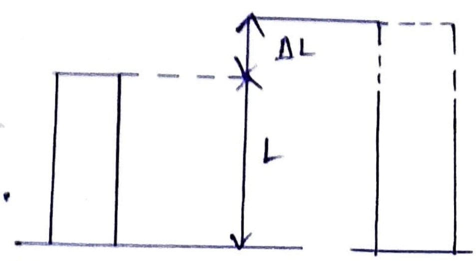
[Fig: THERMAL STRESS]

→ If expansion is prevented, compressive stress are induced.

→ If contraction is prevented, tensile stress are induced.

TEMPRATURE STRAIN:

→ It is the ratio of prevention of expansion or contraction due to change in temperature per unit original dimension.



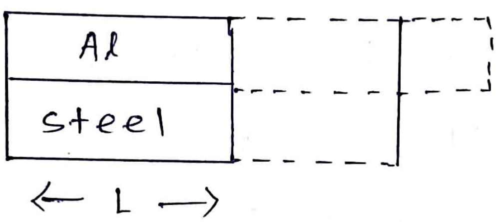
[Fig: Tempreature strain]

→ The natural of stress is compressive, if expansion is prevented.

→ Tensile, if contraction is prevented.

Case - I

Let 2 bars joined face to face subjected to increase in tempreature.



R.I.N

$\Delta L = L \times \alpha T$

$\Delta L_{AL} = 0.04 \times 23 \times 10^{-6} \times 30^{\circ}$

$= 27.6 \times 10^{-6} m$

$\Delta L_{steel} = 0.04$

$\alpha_{AL} = 23 \times 10^{-6} \, {}^{\circ}C$

$\alpha_{steel} = 12 \times 10^{-6} \, {}^{\circ}C$

So, $\alpha_{Al} > \alpha_{steel}$

Letus consider a composite bar made up of 'al' & stell joint face to face as shown in the figure.

We know that,

$\alpha_{Al} > \alpha_{steel}$

But in this composite bar steel bar try to prevent the deformation of Al bar and steel bar try to deform its length to a larger length. this expansion is common for both Steel and Aluminium and they reach to a new common point.

The common expansion of composite bar is less than the free expansion of Aluminium bar. and then more the expansion of steel bar so it can be said that the Al bar is subjected to compressive stress and the steel bar is subjected to tensile stress.

During force equilibrium compressive force in Aluminium = Tensile force in steel.

We know $\sigma = \frac{F}{A}$

$$\Rightarrow F = \sigma A$$

so, $\sigma_{AL} A_{Al} = \sigma_S A_S$

$$(\Delta L \text{ actual})_{AL} = (\Delta L \text{ actual})_{steel}$$

$$\Rightarrow + \left(L \alpha \Delta T - \frac{\sigma_{Al} L}{E_{Al}} \right) = L \alpha_{steel} \Delta T + \frac{\sigma_S L}{E_S}$$

α_{Al} → Co-efficient of thermal expansion of steel.

L → Length of the composite bar.

α_S → Co-efficient of thermal expansion of steel.

A_{Al} → Area of cross section of 'Al' bar

A_S → Area of cross section of steel bar.

E_{Al} → Young's modulus of elasticity of Al bar.

E_S → Young's modulus of elasticity of steel bar.

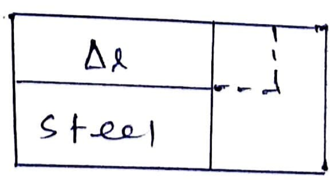
ΔT → Change in temperature.

σ_A → compressive stress in 'Al'

σ_S → Compressive stress in steel.

Case-2

When 2 bar joined face to face subjected to decrease in tempreature.



(Δ_L actual)

Let us consider a composite bar made up of 'al' & stell and joint face to face as shown in the figure.

We know that

$\alpha_{AL} > \alpha_{Steel}$

But in this composite bar steel bar try to prevent the contraction of 'Al' bar and steel bar try to contract its length. This contract is common for both steel and Aluminium and they reach to a new common point.

The common contraction of composite bar is more than the free expansion of 'Al' bar and less than the expansion of steel bar so it can be said that the Aluminium bar is subjected to tensile stress and the steel bar is subjected to compressive stress.

We know that,

$$\sigma = \frac{F}{A}$$

$$\Rightarrow F = \sigma A$$

So,

$$\sigma_A A_A = \sigma_s A_s$$

$$(\Delta L \text{ actual})_{Al} = (\Delta L \text{ actual})_{steel}$$

$$-\left(L\alpha_{Al}\Delta T - \frac{\sigma_{Al} L}{E_{Al}}\right) = L\alpha_s \Delta T + \frac{\sigma_s L}{E_s}$$

LECT-25 & 26

15/03/2021

THERMAL STRESS :—

Total normal stress developed on the c/s = Mechanical Stress + Thermal Stress

Mechanical Stress ↓ can be Normal / Shear

Thermal Stress ↓ Always a Normal Stress

But here we consider only normal Stress (Axial + Bending Stress)

Mech. stress developed due to loads acting on the c/s.

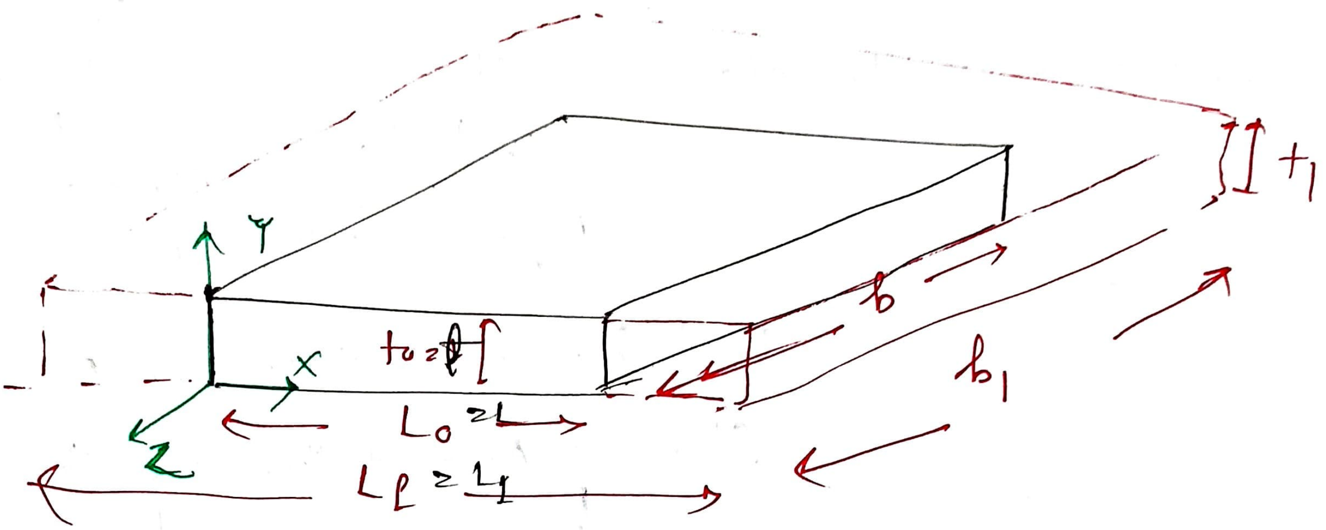
Thermal Stress — (i) there should be a change in temp. variation. ie either thermal expansion or contraction occurs

(ii) Thermal deformation should be restricted either completely or partially

Thermal Stress = 0 when only 1st condition is Satisfied (ie free expansion / contraction)

Nothing is there to restrict its motion.

CASE - 1 → FREE EXPANSION OF A RECTANGULAR BLOCK :—



(Deformed shape of the block due to change in Temp.)

T → change in Temp

$$(\delta_{th})_{xx} = \delta L = L_1 - L = \alpha T L$$

$$(\delta_{th})_{yy} = \delta_t = t_1 - t = \alpha T t$$

$$(\delta_{th})_{zz} = \delta_b = b_1 - b = \alpha T b$$

$$(\delta_{th})_{xx} > (\delta_{th})_{z} > (\delta_{th})_{y}$$

$$\text{Bcoz } (L > b > t)$$

But in case of cube :—

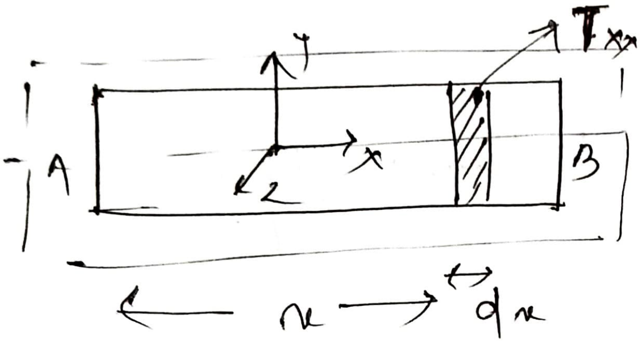
$$(\delta_{th})_x = (\delta_{th})_y = (\delta_{th})_z = \alpha T a$$

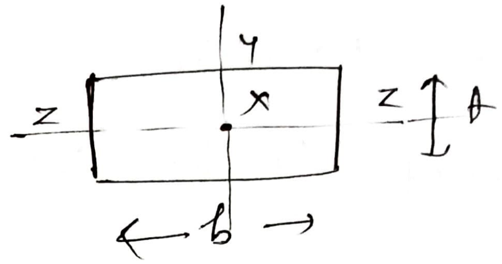
These expression are Valid under following condition

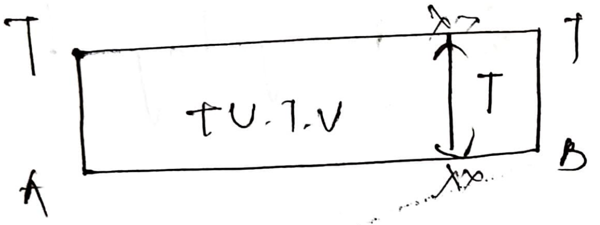
(i) free expansion

(ii) Same Material

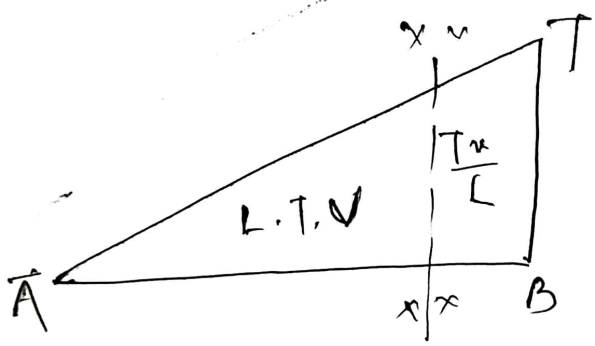
(iii) Uniform Temp Variation (U.T.V)







$$(\delta L)_{strip} = \alpha_{xx}(T_{xx}\,dx)$$



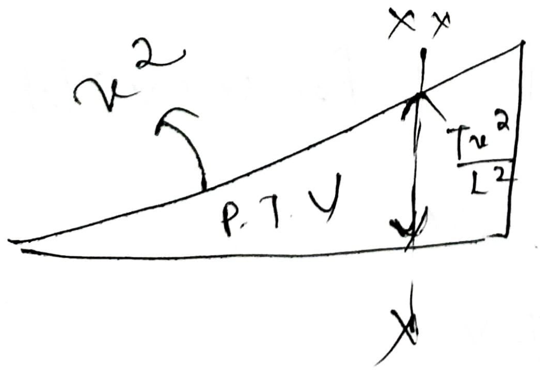
$$(\delta L)_{block} = \int_0^L \alpha (T_{xx})(dx)$$

U.T.V; $(\delta L)_{block} = \alpha T \int_0^L dx$

$= \alpha T L$.

L.T.V; $(\delta L)_{block} = \int_0^L \alpha \left(\frac{Tx}{L}\right) dx$

$$(\delta L)_{block} = \frac{\alpha T L}{2}$$



P.T.V → Parabolic Temp Variation.

P.T.V ; $T_{xx} = \frac{Tx^2}{L^2}$ $\Delta L = \int_0^L \alpha\left(\frac{Tx^2}{L^2}\right) dx$

$(\delta L)_{block} = \frac{\alpha TL}{3}$ $\left(\Delta L = \frac{\alpha T}{L^2}\left(\frac{x^3}{3}\right)\right.$

$\left.\Delta L = \frac{\alpha TL}{3}\right)$

Cubic parabolic Temp Variation (C.P.T.V) ; $T_{xx} = \frac{Tx^3}{L^3}$.

$\therefore (\delta L)_{block} = \frac{\alpha TL}{4}$ by integrating it

$\delta L = (\delta th)_{xx} = \frac{\alpha TL}{K}$

$\delta b = (\delta th)_{zz} = \frac{\alpha Tb}{K}$

$\delta_t = (\delta th)_y = \frac{\alpha Tt}{K}$

where K = constant i.e. K = 1 for U.T.V (default)

K = 2 for L.T.V

K = 3 for P.T.V

→ K = 4 for C.P.T.V

$$(\epsilon_{th})_x = \frac{\delta L}{L} = \frac{\alpha T}{K}$$

$$(\epsilon_{th})_x = (\epsilon_{th})_y = (\epsilon_{th})_z = \pm \frac{\alpha T}{K}$$

Thermal strains in 3 Mutually ⊥r directions are equal & like in nature.

+ve → when ϵ_{th} is Tensile in nature. i.e when Temp ↑ & Vice-Versa

IMP

$$\boxed{(\sigma_{th})_x = (\sigma_{th})_y = (\sigma_{th})_z = 0 \quad \text{Becoz} \because R_x = R_y = R_z = 0}$$

As $E = \dfrac{\sigma_{th}}{\epsilon_{long}}$

$\Rightarrow \sigma_{th} = E\,\epsilon_{long}$ As $\epsilon_{long} = 0$

$\Rightarrow \sigma_{th} = 0$

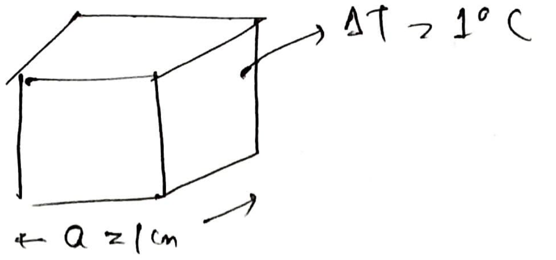
As Thermal stress always depend upon the longitudinal strain.

$$\epsilon_v = \frac{dv}{V} = \epsilon_x + \epsilon_y + \epsilon_z$$

$$= 3\,\epsilon_{th}$$

$$\frac{\delta V}{V} = \pm\left(3\frac{\alpha T}{K}\right)$$

Ex1 -



; $(\delta V)_{cube} = ?$

$\Rightarrow \dfrac{\delta V}{V} = \cancel{3\alpha T}$ $T = 1°C$; $K = 1$ for U.T.Y

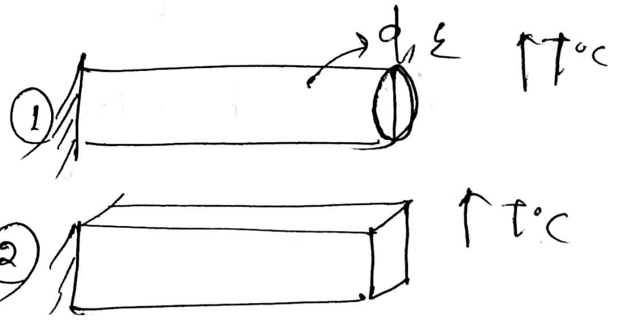
$\Rightarrow \delta V = \cancel{3\alpha T a^3} = 3\alpha\ cm^3$

Thermal deformation in any direction ie $(\delta_{th})_{xx}$ or $(\delta_{th})_{yy}$ or $(\delta_{th})_{zz}$ only depends on the dimensions in that direction & independent of dimensions in other 2 directions.

But Mechanical deformation in any direction depends upon the dimensions in all the 3 directions. (UnEqual & Unlike).

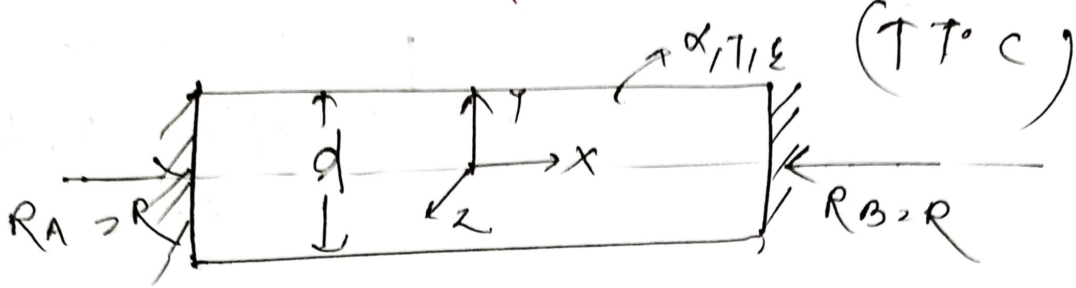
$\Delta L_1 = L\alpha T$

$\Delta L_2 = L\alpha T$

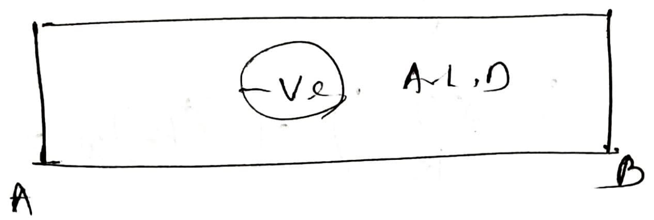


ie the change in Length is independent of ~~other~~ c/s area. So If they are made of same materials same length & ~~free~~ change in Temp is same Then their (ΔL) will remains same.

CASE - II ; COMPLETELY RESTRICTED EXPANSION IN ONE DIRECTION



(Fig: A prismatic bar is rigidly held b/w 2 supports.)



So $P = -R$

$$\sigma_a = (\sigma_{th})_x = \frac{P}{A} = \frac{-R}{A} \quad \text{---(i)}$$

$$(\delta L)_{bar} = (\delta L)_{T.D} + (\delta L)_{A.D} = 0.$$

$$= \left[\frac{\alpha TL}{K}\right] + \left[\frac{(P = -R)L}{AE}\right]$$

$$0 = \frac{\alpha TL}{K} - \frac{RL}{AE}$$

$$\Rightarrow \frac{\alpha TL}{K} = \frac{RL}{AE}$$

$$\Rightarrow R = \frac{\alpha TAE}{K} \quad \text{---(ii)}$$

from eqn ; $(\sigma_a \text{ or } \sigma_{th})_x = -\frac{\alpha TEA}{KA} = \frac{-\alpha TE}{K}$ (Comp)

$$\sigma_a = (\sigma_{th})_x = \frac{\alpha T E}{K} \text{ (Comp)} \quad \text{— (III)}$$

→ P.B
→ Same material
→ Completely Restricted definition

$$\epsilon_{leng} = \epsilon_x = \frac{\sigma_x}{E} = \left(\frac{\sigma}{E}\right) - \frac{\alpha T}{K} \quad \text{— (IV)}$$

$$\epsilon_{TOTAL} = \epsilon_x = (\epsilon_{leng})_x + (\epsilon_{th})_x = 0$$

$$= \left(-\frac{\alpha T}{K}\right) + (\epsilon_{th})_x = 0$$

$$\Rightarrow \boxed{(\epsilon_{th})_x = \frac{\alpha T}{K}} \quad \text{— (V)}$$

$$(\epsilon_{th})_x = (\epsilon_{th})_y = (\epsilon_{th})_z = \frac{\alpha T}{K},$$

ϵ_{th} & ϵ_{leng} ⇒ Are equal & Unlike in nature
(Completely Restricted deformation)

(ϵ_{TOTAL}) y & z direction $= \frac{\delta d}{d} = \epsilon_{th} + (\epsilon_{lateral})_{y \,\&\, z}$

$$\Rightarrow \frac{\delta d}{d} = \left\{\left(\frac{\alpha T}{K}\right) + (-\mu \, \epsilon_{leng})\right\}$$

$$\left(\epsilon_{TOTAL}\right)_{y+z} = \frac{\delta d}{d} = \left\{\frac{\alpha T}{K} - \mu\left(-\frac{\alpha T}{K}\right)\right\}$$

$$\frac{\delta d}{d} = \frac{\alpha T}{K}\left(1+\mu\right)$$

$$\Rightarrow \delta d = \frac{\alpha T d}{K}\left(1+\mu\right)$$

$$\boxed{\sigma_{th} = \pm\frac{\alpha T E}{K}} \qquad \boxed{\epsilon_{th} = \pm\frac{\alpha T}{K}}$$

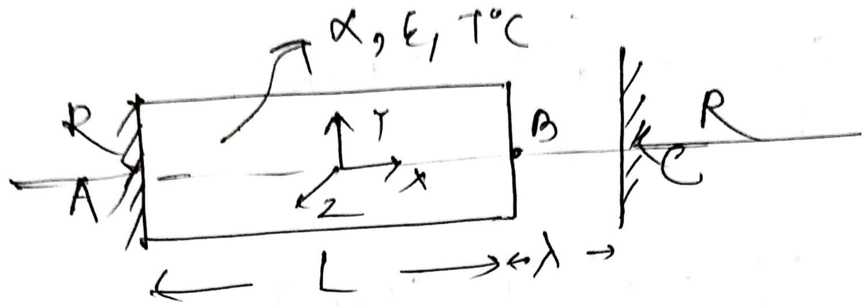
when Temp $T(\uparrow)$; σ_{th} is $-ve$ & ϵ_{th} is $+ve$

ϵ_{long} is $-ve$

when Temp $T(\downarrow)$; σ_{th} is $+ve$; ϵ_{long} is $+ve$

& ϵ_{th} is $-ve$.

CASE-3 PARTIALLY RESTRICTED EXPANSION IN ONE DIRECTION! —



where $\lambda \rightarrow$ change in Length of bar

or Gap b/w bar & supports.

or Gap b/w 2 adjacent rails.

(To avoid buckling; gap provided) In the rail

or yielding of supports. or spring defn

or expansion permitted

$\left(\delta_{th} = \alpha T L - \lambda\right) \Rightarrow$ Restricted Expansion

So $P = -R$

$$\left(\sigma_a = \sigma_{th}\right)_x = \frac{P}{A} = \frac{-R}{A} \quad \text{— (i)}$$

$$(\delta L)_{bar} = (\delta L)_{T.D} + (\delta L)_{A.D} = \lambda$$

$$= \left(\frac{\alpha T L}{K}\right) + \left(\frac{-RL}{AE}\right) = \lambda$$

$$\Rightarrow \frac{RL}{AE} = \frac{\alpha TL}{K} - \lambda$$

$$\therefore R = \frac{AE}{L}\left(\frac{\alpha TL}{K} - \lambda\right)$$

$$(\sigma_a \text{ or } \sigma_{th})_x = \frac{R}{A} = \left[\frac{E}{L}\left(\frac{\alpha TL}{K} - \lambda\right)\right]$$

$$\text{So } (\sigma_a \text{ or } \sigma_{th})_x = \left(\frac{\alpha TL}{K} - \lambda\right)\frac{E}{L} \quad \text{(iii)}$$

→ P.B
→ Same Material

All other remaining condition can be derived from these expression as above

Eq (iii) is valid for free expansion, completely Restricted expansion & partially restricted expansion

For free expansion; $\lambda = \delta L = \frac{\alpha TL}{K}$

$$\therefore \sigma_{th} = 0.$$

For completely Restricted Expansion;

$$\lambda = \delta L = 0$$

$$\Rightarrow \sigma_{th} = -\frac{\alpha TE}{K}$$

(2:35) hrs

Similarly ϵ_{long}, ϵ_{lat} can be derived from this

For free expansion :—

$$(\epsilon_{th})_x = (\epsilon_{th})_y = (\epsilon_{th})_z = (+) \frac{\alpha T}{K}$$

$$\epsilon_{long} = \epsilon_{lateral} = 0$$

$$\sigma_{th} = 0$$

For Comp. restricted expression :—

$$(\epsilon_{th})_x = (\epsilon_{th})_y = (\epsilon_{th})_z = \pm\left(\frac{\alpha T}{K}\right)$$

$$\epsilon_{long} = -(\epsilon_{th})_x = \pm\left(\frac{\alpha T}{K}\right)$$

$$(\sigma_{th})_x = \pm\left(\frac{\alpha T E}{K}\right) ; (\sigma_{th})_y = (\sigma_{th})_z = 0$$

$$\epsilon_{lateral} = -\mu\,\epsilon_{long}$$

For Partially restricted deformation :—

$$(\epsilon_{th})_x = (\epsilon_{th})_y = (\epsilon_{th})_z = \pm\left(\frac{\alpha T}{K}\right)$$

$$\epsilon_{long} \neq -\epsilon_{th}$$

$$(\sigma_{th})_{xx} = \pm\left[\frac{\alpha T L}{K} - \lambda\right]\frac{E}{L}$$

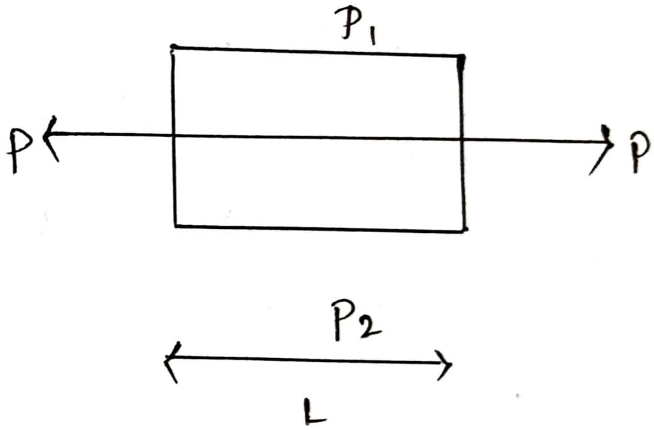
$$\epsilon_{long} = \frac{\sigma_{th}}{E} = \pm\left[\frac{\alpha T L}{K} - \lambda\right]\frac{1}{L}$$

Lec

Deformation in composite bar :-

$$P = P_1 + P_2 \quad \text{---------} (1)$$

$$\Delta_1 = \Delta_2$$



$$\Rightarrow \frac{P_1 \cancel{L}}{A_1 E_1} = \frac{P_2 \cancel{L}}{A_2 E_2}$$

$$\Rightarrow \frac{P_1}{A_1 E_1} = \frac{P_2}{A_2 F_2}$$

$$\Rightarrow P_1 = P_2 \left(\frac{A_1 E_1}{A_2 E_2} \right) \quad \text{---------} (11)$$

Putting the value of P_1 in $eq^{(n)}$ (i)

$$P = P_2 \left(\frac{A_1 E_1}{A_2 E_2} \right) + P_2$$

$$P = P_2 \left\{ \left(\frac{A_1 E_1}{A_2 E_2} \right) + 1 \right\}$$

$$P_2 = \frac{P}{\frac{A_1 E_1 + A_2 E_2}{A_2 E_2}}$$

$$\Rightarrow P_2 = \frac{P(A_2E_2)}{A_1E_1 + A_2E_2}$$

So $P = P_1 + P_2$

$$P = P_2\left(\frac{A_1E_1}{A_2E_2}\right) + P\left(\frac{A_2E_2}{A_1E_1 + A_2E_2}\right)$$

STRAIN ENERGY :-

It is defined as the stored energy within the body to give resistance to an externally applied force when an externa load is applied on a member it gets deform but the body develops some resistance against the deformation and this resistance is stored within the body in the form of energy. which is called strain energy.

work done / strain energy

$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$

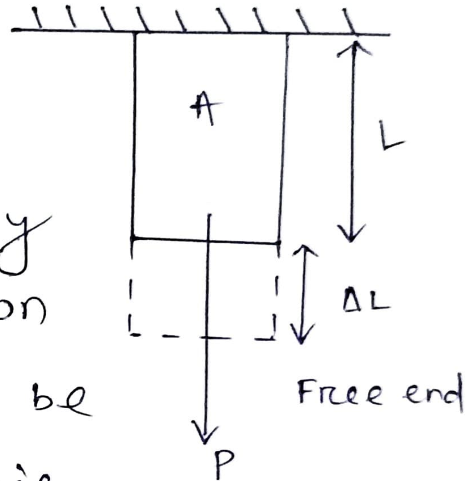
$= \frac{1}{2} \times \sigma \times \epsilon \times V$

$= \frac{1}{2}\sigma \times \frac{\sigma}{E} \times V$

$$\Rightarrow \boxed{S.E = \frac{\sigma^2}{2E}(V)}$$

STRESS DUE TO GRADUALLY APPLIED LOAD :-

Lets consider a uniform bar subjected to an axial tensile load (P) as shown in the figure at the free end the bar is fix at another end.



Due to the force application some amount of deformation will occure with in the body to resistance this deformation some internal resistance will be develope with in the body this resistance will be stored in the form of energy within the body which is called strain energy.

so, work done / strain energy $= \frac{1}{2} \sigma \times \epsilon \times V$

$$= \frac{1}{2} \sigma \times \frac{\sigma}{E} \times V$$

$$= \frac{\sigma^2}{2E} (V)$$

$$W = \frac{\sigma^2}{2E} (\Delta L) \text{ ——— } (1)$$

As the load is gradually applied from 0 to P. so the work done can be written as..

Work done = Avg load × Distance

$$= \frac{1}{2}(0+P) \times \Delta L$$

$$= \frac{P}{2} \times \Delta L$$

$$W = \frac{P}{2} \times \frac{\sigma L}{E} \quad \text{——— (2)}$$

solving $eq^{(n)}$ (i) & (ii)

$$\frac{\sigma^2}{\cancel{2}\cancel{E}}(A\cancel{L}) = \frac{P}{\cancel{2}} \times \frac{\sigma \cancel{L}}{\cancel{E}}$$

We know that

$$\sigma \propto \epsilon$$

$$\sigma = E\epsilon$$

$$\Rightarrow \epsilon = \frac{\sigma}{E}$$

$$\Rightarrow \frac{\Delta L}{L} = \frac{\sigma}{E}$$

$$\Rightarrow \Delta L = \frac{\sigma L}{E}$$

$$\Rightarrow \sigma A = P$$

$$\Rightarrow \boxed{\sigma = \frac{P}{A}}$$

where,

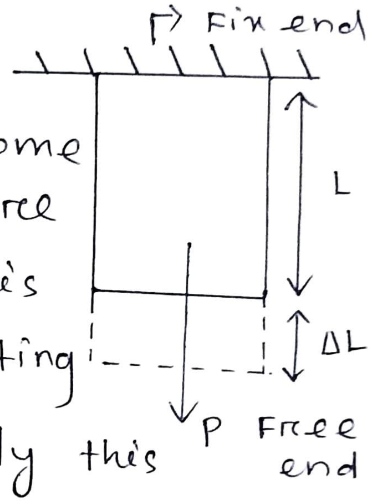
'A' - Area of cross section of the bar.

'L' - Length of the bar

'ΔL' - Deformation of the bar due to the application of the tensile load.

STRESS DUE TO SUDDENLY APPLIED LOAD:-

Letus consider a uniform bar subjected to axial tensile load (p) as shown in the figure at the free end the bar is fixed at the another end.



Due to the force application some amount of deformation will occure with in the body to resist this deformation some internal resisting will be develop with in the body this resistance will be stored in the form of energy within the body which is called as strain energy.

Work done/ strain energy:-

$$= \frac{1}{2} \times \sigma \times \epsilon \times V$$

$$= \frac{1}{2} \sigma \times \frac{\sigma}{E} \times V$$

$$= \frac{\sigma^2}{2E} \times V$$

$$N = \frac{\sigma^2}{2E} \times (AL) \quad \text{———} \quad (1)$$

As a suddenly applied load P is acting on this vertical bar.

$$\text{Work done} = P \times \Delta L$$

$$= \frac{P\sigma L}{E}$$

Equating eq$^{(n)}$ (1) and (2) we get,

$$\frac{\cancel{\sigma}^2}{2\cancel{E}}(A\cancel{L}) = P\left(\frac{\cancel{\sigma}\cancel{L}}{\cancel{E}}\right)$$

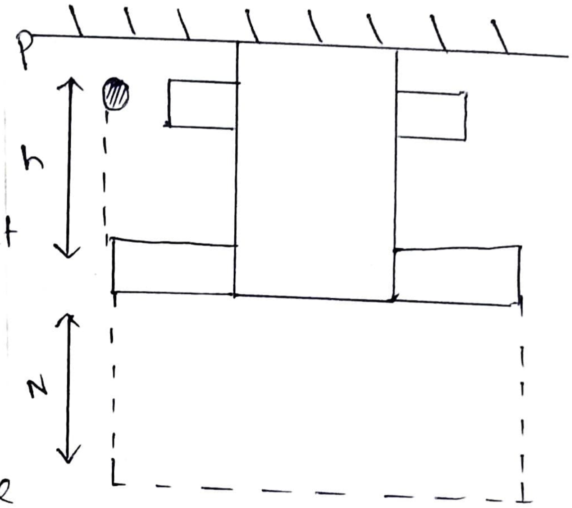
$$\frac{\sigma A}{2} = P$$

$$\Rightarrow \sigma A = 2P$$

$$\Rightarrow \boxed{\sigma = \frac{2P}{A}}$$

$$\boxed{\sigma_s = 2\sigma_G}$$

STRESS DUE TO IMPACT LOAD :-



Let us consider a vertical bar which is fix at one end as shown in the figure and an Impact load of amount 'p' is droped from a certain hight 'h' as shown in the figure and due to this impact load is the deformation of the vertical bar will become ΔL. Due to this Application OF LOAD the body will develop some resistance and this resistance will be stored in the form of energy. i.e :- Known as strain Energy.

So,

$$\text{Work done} / S.E = \frac{1}{2}\sigma \times e \times V$$

$$= \frac{1}{2}\sigma \times \frac{\sigma}{E} \times V$$

$$= \frac{\sigma^2}{2E}(AL) \quad \text{———— (i)}$$

workdone against the distance covered

$$W = P \times (h \times \Delta L)$$

$$= P\left(h + \frac{\sigma L}{E}\right) \text{———} (ii)$$

(According to Hooke's law)

Equating eq$^{(n)}$ (i) and (ii) we get,

$$\frac{\sigma^2}{2E}(AL) = P\left(h + \frac{\sigma L}{E}\right)$$

$$\Rightarrow \sigma^2\left(\frac{AL}{2E}\right) = Ph + P\left(\frac{\sigma L}{E}\right)$$

$$\Rightarrow \sigma^2\left(\frac{AL}{2E}\right)\left(\frac{2E}{AL}\right) = \frac{2E}{AL}(Ph) + \frac{2E}{AL}\left(\frac{P\sigma L}{E}\right)$$

$$\Rightarrow \sigma^2 = \frac{2EPh}{AL} + 2\left(\frac{P}{A}\right)\sigma$$

$$\Rightarrow \sigma^2 - 2\sigma\left(\frac{P}{A}\right) = \frac{2EPh}{AL}$$

$$\Rightarrow \sigma^2 = 2\sigma\left(\frac{P}{A}\right) + \left(\frac{P}{A}\right)^2$$

$$= \left(\frac{P}{A}\right)^2 + \frac{2EPh}{AL}$$

$$\Rightarrow \left(\sigma - \frac{P}{A}\right)^2 = \left(\frac{P}{A}\right)^2 + \frac{2EPh}{AL}$$

$$\Rightarrow \left(\sigma - \frac{P}{A}\right) = \pm\sqrt{\left(\frac{P}{A}\right)^2 + \frac{2EPh}{AL}}$$

$$\Rightarrow \boxed{\sigma = \frac{P}{A} \pm \left(\frac{P}{A}\right)^2 + \frac{2EPh}{AL}}$$

Q.1 A steel rode of 25 m.m diameter and length is 4 m is subjected an Axial pull of 40 KN then determine.

(I) Intensity of stress

(II) Strain

(III) change in length or deformation of rode.

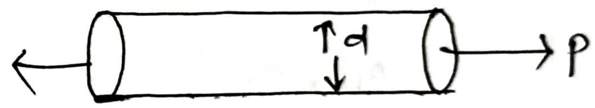
Sol$^{(n)}$:- Given that

$d = 25$ mm

$L = 4$ m

$P = 40$ KN

$E = 0.21 \times 10^6$ N/mm^2.



so,

$$A = \frac{\pi}{4} d^2$$

$$= \frac{\pi}{4} (25)^2$$

$$= 490.87 \text{ mm}^2$$

$$\sigma = \frac{P}{A} = \frac{40 \times 10^3}{490.87 \text{ mm}^2}$$

$$= 81.48 \text{ Mega pasc}$$

We know that

$$\sigma = E\epsilon$$

$$\Rightarrow \epsilon = \frac{\sigma}{E}$$

$$\epsilon = \frac{81.49 \text{ N/mm}^2}{0.21 \times 10^6 \text{ N/mm}^2}$$

$$= 388.05 \times 10^{-6}$$

Given,

$$\epsilon = 0.38 \times 10^{-3}$$

$$\Rightarrow \frac{\Delta L}{L} = 0.38 \times 10^{-3}$$

$$\Rightarrow \Delta L = 0.38 \times 10^{-3} \times L$$

$$= 0.38 \times 10^{-3} \times 4000$$

$$\Rightarrow \Delta L = 1.5 \text{ mm}.$$

Q.2 A steel rod of 10 m.m diameter and 300 m.m length alonget by 0.18 mm under are axial Pull of 10 K.N then determine the Young's Modulus of elastic of a material.

Sol(n) ÷ Given that

$d = 10$ m.m

$L = 300$ mm

$P = 10$ K.N

$\Delta L = 0.18$ mm

$$\sigma = \frac{P}{A} = \frac{10\ KN}{\frac{\pi}{4}(d)^2} = \frac{10 \times 10^3\ N}{\frac{\pi}{4}(10)^2\ mm^2} = \frac{10 \times 10^3\ N}{78.53\ mm^2}$$

$$= 127.33\ N/mm^2$$

$$\epsilon = \frac{\Delta L}{L} = \frac{0.18\ mm}{300\ mm} = 6 \times 10^{-4}$$

$$E = \frac{\sigma}{\epsilon} = \frac{127.33}{6 \times 10^{-4}}$$

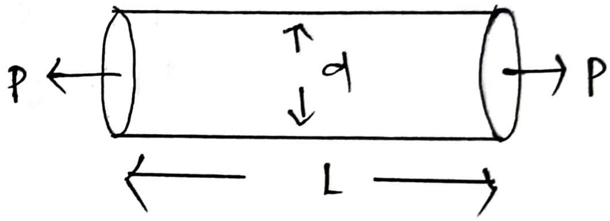
$$= 0.212 \times 10^6\ N/mm^2$$

Q.3 A mild steel rode of 10 m.m diameter of 500mm length. He subjected to a tensile load of 20 K.N. It alongated by 0.15 mm by the application of the load then find out

(I) Intensity of stress

(II) Strain

(III) Modulus of elasticity.



Sol$^{(n)}$:- Given that

$d = 10\,mm$

$L = 500\,mm$

$P = 20\,KN$

$\Delta L = 0.15\,mm.$

(i) We know that

$$\sigma = \frac{P}{A} = \frac{20 \times 10^3 N}{\left(\frac{\pi}{4}\right)(10)^2 mm^2}$$

$$= \frac{20 \times 10^3 N}{78.53\,mm^2}$$

$$= 254.6\ N/mm^2$$

(ii) $\epsilon = \Delta L / L$

$$= 0.15\,\cancel{m}/\,500\,\cancel{m}$$

$$= 3 \times 10^{-4}$$

(iii) We know that

$$\sigma = E\epsilon$$

$$E = \frac{\sigma}{\epsilon} = \frac{254.6}{3 \times 10^{-4}}\ N/mm^2$$

$$= 84.64 \times 10^4\ N/mm^2$$

Q.(4) A rectangular steel bar 60 m.m wide and 10 mm thick and 3m long under an axial pull the increase in length is 1.5 m and decrease in thickness 0.0014 m.m calculate the poision's ratio of the bar?

Sol$^{(n)}$:- Given that

$$L = 3m = 3000 \text{ mm}$$

$$\Delta L = 1.5 \text{ m} = 1500 \text{ m.m}$$

$$T = 10 \text{ m.m}$$

$$\Delta T = 0.0014 \text{ mm}$$

We know that

$$\epsilon_L = \frac{\Delta L}{L} = \frac{1500}{3000} = \frac{1}{2} = 0.5$$

$$\epsilon_T = \frac{\Delta T}{T} = \frac{0.0014}{10} = 1.4 \times 10^{-4}$$

Again we know that

$$\text{poissions ratio} = \frac{\text{Lateral strain}}{\text{longitudinal strain}}$$

$$= \frac{1.4 \times 10^{-4}}{0.5}$$

$$= 2.8 \times 10^{-4}$$

(Ans)

Q.(5) A steel bar 2.4 m long and 20 m.m in diameter It was stretched by 1.2 mm under an axial pull of 32 KN Determine the stress, strain young's modulus and modulus of rigidity take $1/m = 0.25$

Sol(n) Given that

$$L = 2.4\text{ m} = 2.4 \times 10^3\text{ mm}$$

$$\Delta L = 1.2\text{ mm}$$

$$d = 20\text{ mm}$$

$$P = 32\text{ KN} = 32 \times 10^3\text{ N}$$

We know that

$$\sigma = \frac{P}{A} = \frac{32 \times 10^3}{\frac{\pi}{4}(20)^2} = 1018.69\text{ Mpa (Ans)}$$

$$\epsilon = \frac{\Delta L}{L} = \frac{12}{2.4 \times 10^3} = 0.5 \text{ (Ans)}$$

$$E = \frac{\sigma}{\epsilon} = \frac{1018.59}{0.5} = 2037.18\text{ Mpa}$$

(Ans)

We know that

$$E = 2G(1+\mu)$$

$$\Rightarrow 2037.18 = 2G(1+\mu)$$

$$\Rightarrow 2037.18 = 2G(1+0.25)$$

$$\Rightarrow 2037.18 = 2G + 0.5G$$

$$\Rightarrow 2037.18 = G(2+0.5)$$

$$\Rightarrow G = \frac{2037.18}{2.5} \Rightarrow G = 814.872\text{ MPa}$$

Q.6 A barr of 10 mm x 10 mm size and 400 m.m long is subiected to an axial pull of 12 KN. the elongation in length the contruction in lateral dimension are found to be 0.4 mm and 0.0025 m.m respectively. Determine poision ratio and young's modulus, Modulus of rigidity and bulk modulus of material.

Sol$^{(n)}$:- Given that,

$$A = 10 \times 10 = 100 \, mm^2$$

$$l = 400 \, mm$$

$$\Delta l = 0.4 \, mm$$

$$P = 12 \, KN = 12 \times 10^3 \, N$$

Lateral dimension = 0.0025.

We know that,

$$\sigma = \frac{P}{A} = \frac{12 \times 10^3}{100} = 120 \, N/mm^2$$

$$\epsilon = \frac{\Delta L}{L} = \frac{0.4}{400} = 0.001$$

$$E = \frac{\sigma}{\epsilon} = \frac{120}{0.001} = 120000 \, N/mm^2$$

$$= 12 \times 10^4 \, N/mm^2$$

$$\text{Lateral strain} = \frac{\Delta b}{b}$$

$$= \frac{0.0025}{10} = 2.5 \times 10^{-4}$$

$$\text{Poission ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{2.5 \times 10^{-4}}{0.001}$$

$$= 0.25$$

Modulus of Rigidity $E = 2G(1+v)$

$$12 \times 10^4 = 2.G(1+0.25)$$

$$\Rightarrow 12 \times 10^4 = G(2.50)$$

$$\Rightarrow G = \frac{12 \times 10^4}{2.50} = 4.8 \times 10^4 \ N/mm^2$$

Bulk's Modulus of material

$$E = 3K(1-2v)$$

$$\Rightarrow 12 \times 10^4 = 3K(1-2.0.25)$$

$$\Rightarrow 12 \times 10^4 = 3K(1-0.5)$$

$$\Rightarrow 12 \times 10^4 = K(3-1.5)$$

$$\Rightarrow K = \frac{12 \times 10^4}{1.5}$$

$$= 8 \times 10^4 \ N/mm^2$$

Q.(07) A bar of 30 c.m dia is subjected to a pull of 60 KN. The measured extension over. length of 200 mm. is 0.09 mm and change in dia is 0.0039 mm. Find the poistions ratio and values of the modul.

Sol$^{(n)}$:

$d = 30$ c.m $= 300$ mm

$P = 60$ K.N $= 60 \times 10^3$ N

$l = 200$ mm

$\Delta l = 0.09$ mm

$\Delta d = 0.0039$ mm

We know that

$$\epsilon_L = \frac{\Delta L}{L} = \frac{0.09}{200} = 4.5 \times 10^{-4}$$

$$\epsilon_d = \frac{\Delta d}{d} = \frac{0.0039}{300} = 1.3 \times 10^{-5}$$

$$v = \frac{\epsilon_d}{\epsilon_L} = \frac{1.3 \times 10^{-5}}{4.5 \times 10^{-4}} = 0.28 \times 10^{-1} = 0.028$$

$$\text{Young's Modulus} = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{Stress} = \frac{P}{A} = \frac{60 \times 10^3}{\frac{\pi}{4}(30)^2} = \frac{60000}{706.85} = 84.88 \, N/mm^2$$

$$\text{strain} = 4.5 \times 10^{-4}$$

$$\therefore E = \frac{84.88}{4.5 \times 10^{-4}} = 18.86 \times 10^4 \, N/mm^2$$

Shear Modulus $E = 2G(1+v)$

$$\Rightarrow 18.86 \times 10^4 = (1+0.026)2G$$

$$\Rightarrow 18.86 \times 10^4 = G(2+0.056)$$

$$\Rightarrow G = \frac{18.86 \times 10^4}{2.056} = 9.17 \times 10^4 \ N/mm^2$$

Bulk Modulus $\quad E = 3K(1-2\nu)$

$$\Rightarrow 18.86 \times 10^4 = 3K(1-2 \cdot 0.028)$$

$$\Rightarrow 18.86 \times 10^4 = K(3-0.168)$$

$$\Rightarrow K = \frac{18.86 \times 10^4}{2.832}$$

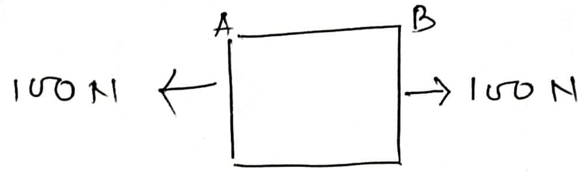
$$= 6.65 \times 10^4 \ N/mm^2$$

Q(08) Find out the total deformation in a mild steel bar of cross sectional area 200 mm^2 and 3.5 m long. The mild steel bar is subjected to different load and different section as shown in the figure. Hence find the total deformation using the principle of superposition.

<u>Sol(n)</u>

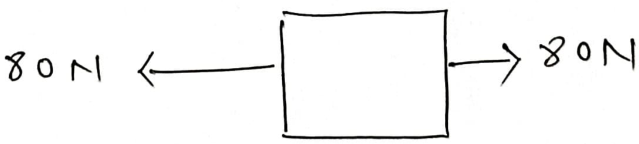
$E = 400\ N/mm^2$

In AB



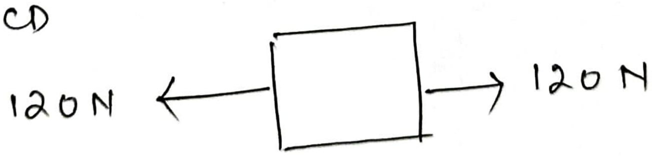
$P_1 = 100\ N$

In BC



$P_2 = 80 N$

In CD



$P_3 = 120 N$

So the total deformation

$$\Delta L = \frac{P_1 L_1}{AE} + \frac{P_2 L_2}{AE} + \frac{P_3 L_3}{AE}$$

$$= \frac{1}{AE}\left(P_1 L_1 + P_2 L_2 + P_3 L_3\right)$$

$$= \frac{1}{200 \times 400} (100 \times 0.5 + 80 \times 1 + 120 \times 2)$$

$$= \frac{1}{8 \times 10^4} (50 + 80 + 240)$$

$$= \frac{370}{8 \times 10^4} = \frac{37 \times 10^{-3}}{8}$$

$$= 4.627 \times 10^{-3} \text{ m}.$$

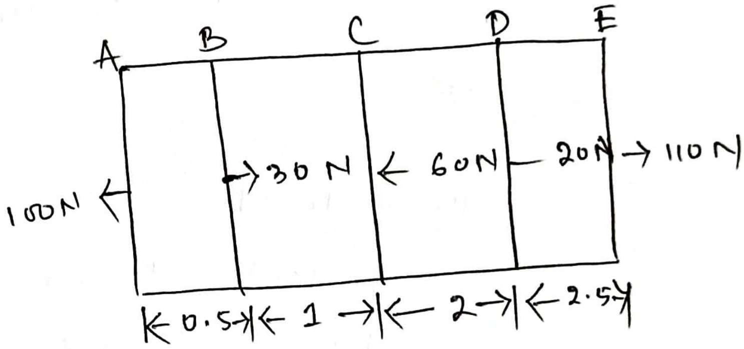
Q.(09) Find out the total deformation of a bar using Principle of superposition of the bar has cross section area of 100 mm^2 and 6m long the bar loaded as shown in the figure take Young's Modulus of elasticity $E = 500\ N/mm^2$

Sol$^{(n)}$:

Given that

$E = 500\ N/mm^2$

$A = 100\ mm^2$



In AB $P_1 = 100\ N$

In BC $P_2 = 70\ N$

In CD $P_3 = 130\ N$

In DE $P_4 = 110\ N$

So the total deformation

$$\Delta L = \frac{P_1 L_1}{AE} + \frac{P_2 L_2}{AE} + \frac{P_3 L_3}{AE} + \frac{P_4 L_4}{AE}$$

$$= \frac{1}{AE}\left(P_1 L_1 + P_2 L_2 + P_3 L_3 + P_4 L_4\right)$$

$$= \frac{1}{100 \times 500}\left(100 \times 0.5 + 70 \times 1 + 130 \times 2 + 110 \times 2.5\right)$$

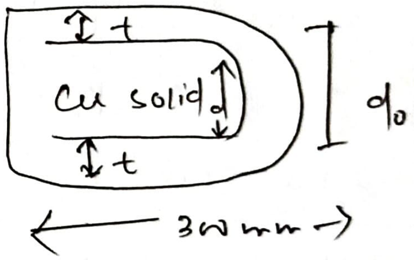
$$= \frac{1}{5 \times 10^4}\left(50 + 70 + 260 + 275\right)$$

$$= \frac{1}{5 \times 10^4}\left(655\right)$$

$$= \frac{655}{5 \times 10^4} = 131 \times 10^{-4}\ m$$

Q.(10) A steel tube of 45 mm diameter and 3 mm thick encloses a solid copper bar of 30 mm diameter the bar and the tube are rigidly connected together at there end at a temp. of 300°K find the stress in each metal i.e steel tube and the copper bar when it is heated to 450°K Find the increasing in length If the original length of the assemble is 300 mm.

Sol^(n): Given that



$d_{steel} = 45\,mm$

$d_{Cu} = 30\,mm$

$t = 3\,mm$

$\alpha_{steel} = 10.8 \times 10^{-8}/°K$

$E_s = 210\,KN/mm^2$

$\alpha_{cu} = 17 \times 10^{-6}/°K$

$E_{Cu} = 110\,KN/mm^2$

$L = 300\,mm$

$\Delta T = 450° - 300° = 150°K$

$d_i = d_o - 2t$

$= 45 - 2 \times 3$

$= 45 - 6 = 39\,mm$

As $\alpha_{cu} > \alpha_{steel}$

Hence in this composite section

copper bar is subjected to compressive stress

steel bar is subjected to tensile stress.

under force of equilibrium.

compressive force in cu bar = Tensile force in steel

i.e

$$\sigma_c A_c = \sigma_s A_s$$

$$\Rightarrow \frac{\sigma_c}{\sigma_s} = \frac{A_s}{A_c}$$

so, $A_{steel} = \frac{\pi}{4}(d_o^2 - d^2)$

$$= \frac{\pi}{4}(45^2 - 39^2)$$

$$= \frac{\pi}{4}(2025 - 1521)$$

$$= \frac{\pi}{4}(504)$$

$$= 395.84 \ mm^2$$

$$A_{cu} = \frac{\pi}{4}(30)^2$$

$$= 706.85 \ mm^2$$

so, $\frac{\sigma_c}{\sigma_s} = \frac{A_s}{A_c} = \frac{395.84}{706.85} = 0.55 \ mm^2$

$$\Rightarrow \sigma_c = 0.55 \ \sigma_s \quad \text{(i)}$$

$$(\Delta L)_{act} = 0$$

$$\Rightarrow [(\Delta L)_{act}]_{cu} + [(\Delta L)_{act}]_{steel} = 0$$

$$\Rightarrow \cancel{L} \alpha_c \Delta T - \frac{\sigma_c \cancel{L}}{E_c} + L \alpha_s \Delta T + \frac{\sigma_s . \cancel{L}}{E_s} = 0$$

$$\Rightarrow \alpha_c \Delta T - \frac{\sigma_c}{E_c} + \alpha_s \Delta T + \frac{\sigma_s}{E_s} = 0$$

$$\Rightarrow (\alpha_c + \alpha_s)\Delta T - \frac{\sigma_c}{E_c} + \frac{\sigma_s}{E_s} = 0$$

$$\Rightarrow (10.8 + 17)10^{-6} \times 150 - \frac{0.55}{110 \times 10^3} + \frac{\sigma_s}{210 \times 10^3} = 0$$

$$\Rightarrow 27.8 \times 150 \times 10^{-6} - \sigma_s \left(\frac{0.55}{110 \times 10^3} + \frac{1}{210 \times 10^3} \right) = 0$$

$$\Rightarrow 4170 \times 10^{-6} = \sigma_s \left(\frac{0.55 \times 10^{-3}}{110} + \frac{10^{-3}}{210} \right)$$

$$\Rightarrow 4170 \times 10^{-6} = \sigma_s \left(5 \times 10^{-6} + 4.76 \times 10^{-6} \right)$$

$$\Rightarrow \sigma_s = \frac{4170 \times 10^{-6}}{10^{-6}(5 + 4.76)}$$

$$\Rightarrow \sigma_s = \frac{4170}{9.76} = 427.25 \text{ N/mm}^2$$

So, $\sigma_c = 0.55\,\sigma_s$

$= 0.55 \times 427.25$

$= 234.98 \text{ N/mm}^2$

Q.(11) Find out the stress due to sudenly applied load when load acting on the body is 300 KN and the area of cross section is 300 mm^2.

Sol(n) ÷ Given that

$P = 300\ KN$

$= 300 \times 10^3 N$

$A = 300\ mm^2$

We know that,

$$\sigma = \frac{2P}{A} = \frac{2.300 \times 10^3}{300}$$

$$= 2 \times 10^3\ N/mm^2.$$

Q.(12) Find out the stress in Mpa when the load acting on a vertical bar is 250 KN and the bar has a circular diameter of 45 mm assume the's load is applied gradually from 0 to 250 K.N.

Sol(n) ÷ Given that

$P = 250\ KN = 250 \times 10^3 N$

$d = 45\ mm$

So, $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(45)^2$

$= 1590.43\ mm^2$

We know that $\sigma = \frac{P}{A}$

$$= \frac{250 \times 10^3}{1590.43}$$

$$= 0.15 \times 10^3\ N/mm^2$$

$$= 150\ N/mm^2\ mpa$$

Q.(13) What is the relation between stress due to gradually applied load (σ_G) & stress due to suddenly applied load (σ_S)?

Sol$^{(n)}$:- We know that

$$\sigma_G = \frac{P}{A}$$

$$\sigma_S = \frac{2P}{A}$$

so $$\boxed{\sigma_S = 2\sigma_G}$$

Q.(14) Find out the stress due to on Impact load of 40 KN when falls from a height of 12 mm on a bar having cross sectional Area 200 mm² the length of the bar is 150 mm & young's modulu's of elasticity E = 250 Mpa.

Sol$^{(n)}$:- Given that

$$P = 40\,KN = 40 \times 10^3 N$$

$$h = 12\ mm$$

$$A = 200\ mm^2$$

$$L = 150\ mm$$

$$E = 250\ Mpa = 250 \times 10^6\ Pa$$

We know that

$$\sigma = \frac{P}{A} \pm \sqrt{\left(\frac{P}{A}\right)^2 + \left(\frac{2EPh}{AL}\right)}$$

$$= \frac{30 \times 10^3 N}{200\ mm^2} \pm \sqrt{\left(\frac{30 \times 10^3 N}{200\ mm^2}\right)^2 + \frac{2 \times 250 \times 30 \times 10^3 \times 12}{200 \times 150}}$$

$$= 150\,\text{Mpa} \pm \sqrt{(150\,\text{Mpa})^2 + 6000\,\text{Mpa}^2}$$

$$= 150\,\text{Mpa} \pm \sqrt{22500\,\text{Mpa}^2 + 6000\,\text{Mpa}^2}$$

$$= 150\,\text{Mpa} \pm \sqrt{28500\,\text{Mpa}^2}$$

$$= 150\,\text{Mpa} \pm 168.81\,\text{Mpa}$$

Ans :-

(i) 318.81 Mpa

(ii) -18.81 Mpa

THIN CYLINDER

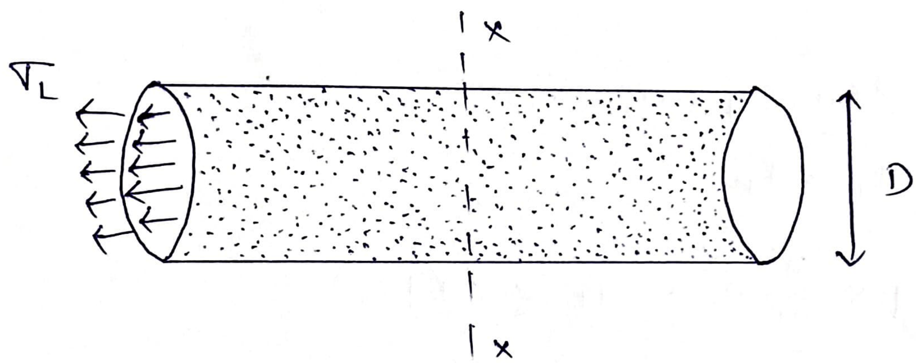
→ A cylinder is said to be a thin cylinder depending on its diameter to its thickness ratio.

→ In thin cylinder we show mainly 2 types of stress.

(i) Longitudinal stress.

(ii) Circumferential stress or Hoop stress.

LONGITUDINAL STRESS :- (σ_L)



Let us consider a thin cylindrical shell which is kept horizontally as shown in the figure. Let a section x-x as given in this figure.

As section x-x divide this horizontally cylinder into two halfs so we can see the pressure force and the brusting force acting on the cylinder. Its the cylinder is storing some amout of fluid hence this fluid exerts fluid pressure 'p'.

i.e :- known as pressure force or brusting force to prevent the brusting force resisting force is be develop along the circular section of the cylinder to keep the cylinder in equilibrium.

Consider the equilibrium of brusting force and the resisting force we will see the following expression.

$$\frac{F_B}{F_R} = \frac{P \times \frac{\pi}{4} D^2}{\sigma_L \times \pi D t} \qquad \because P = \frac{F}{A} \Rightarrow F = PA$$

$$\because A = 2\pi r t$$
$$= \pi D t \qquad c = 2\pi r$$

so under equilibrium ÷

$$F_B = F_R$$

$$\Rightarrow P \times \frac{\cancel{\pi}}{4} D^{\cancel{2}} = \sigma_L \times \cancel{\pi} \cancel{D} t$$

$$\Rightarrow \frac{PD}{4} = \sigma_L t$$

$$\Rightarrow \boxed{\sigma_L = \frac{PD}{4t}}$$

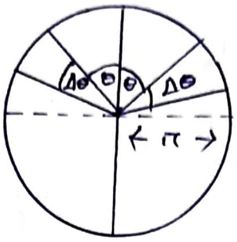
where 'σ_L' is longitudinal stress develop in the cylinder

'p' is fluid pressure exerted on the cylinder.

't' is thickness of the thin cylinder.

'D' is diameter of the thin cylinder.

Hoop stress or circumference stress :-



Lets consider a thin cylinder which is in the Figure. Now Let us consider two elemental strips which is making an angle 'θ' with vertical axis y-y In the elemental strip the angle induced with in the elemental strip is 'Δθ' .

The normal pressure acting on the elemental strip dpn and the normal pressure force acting on the two elemental strips is dfn .

We know that,

$$P = \frac{F}{A}$$

Where force is very small force i.e. dfn .

so, $P = \frac{dfn}{A}$

$\Rightarrow dfn = P \times A$

$\Rightarrow dfn = P \times Lr\Delta\theta$

$F_{Total} = 2 dfn \cos\theta$

$= 2 PLr\Delta\theta . \cos\theta$

$$\int_0^F dF_{Total} = \int_0^F 2PLr\Delta\theta \cdot \cos\theta$$

$$= 2PLr \int_0^F \cos\theta \cdot \Delta\theta$$

$$= 2PLr \left[\sin\theta\right]_0^{\pi/2}$$

$$= 2PLr$$

$$\boxed{F_B = F_{Total} = PLD}$$

Again $F_R = \sigma_H \times A$

$$= \sigma_H \times 2Lt$$

under equilibrium condition

$$F_B = F_R$$

$$\Rightarrow P\cancel{L}D = \sigma_H \times 2\cancel{L}t$$

$$\Rightarrow \boxed{\sigma_H = \frac{PD}{2t}}$$

so, $\sigma_L = \dfrac{\sigma_H}{2}$

$$\Rightarrow \boxed{\sigma_H = 2\sigma_L}$$

Some important formula ÷

$$\epsilon_H = \frac{\sigma_H}{E} - \mu \frac{\sigma_L}{E} \qquad \epsilon_L = \frac{\sigma_L}{E} - \mu \frac{\sigma_H}{E}$$

$$\epsilon_H = \frac{PD}{2tE} - \mu \frac{PD}{4tE} \qquad \epsilon_L = \frac{PD}{4tE} - \mu \frac{PD}{2tE}$$

$$\boxed{\epsilon_H = \frac{PD}{4tE}(2-\mu)} \qquad \boxed{\epsilon_L = \frac{PD}{4tE}(1-2\mu)}$$

'ϵ_H' Hoop strain 'ϵ_L' Longitudinal strain

CHANGE IN DIAMETER ÷

$$\epsilon_H = \frac{PD}{4tE}(2-\mu)$$

$$\Rightarrow \frac{\Delta D}{D} = \frac{PD}{4tE}(2-\mu)$$

$$\Rightarrow \boxed{\Delta D = \frac{PD^2}{4tE}(2-\mu)}$$

CHANGE IN LENGTH ÷

$$\epsilon_L = \frac{PD}{4tE}(1-2\mu)$$

$$\Rightarrow \frac{\Delta L}{L} = \frac{PD}{4tE}(1-2\mu)$$

$$\Rightarrow \boxed{\Delta L = \frac{PDL}{4tE}(1-2\mu)}$$

Volumetric strain :

$$V = \frac{\pi}{4} D^2 (L)$$

$$\Rightarrow \Delta V = \frac{d}{\Delta D}\left(\frac{\pi}{4} L (D^2)\right)$$

$$\Rightarrow \Delta V = \frac{\pi}{4} L (2D\Delta D) + \frac{\pi}{4} D^2 (\Delta L)$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{\cancel{\frac{\pi}{4}} \cancel{L} (2\cancel{D}\Delta D)}{\cancel{\frac{\pi}{4}} D^{\cancel{2}} \cancel{L}} + \frac{\cancel{\frac{\pi}{4}} \cancel{D^2} \Delta L}{\cancel{\frac{\pi}{4}} \cancel{D^2} L}$$

$$\Rightarrow \epsilon_V = 2 \frac{\Delta D}{D} + \frac{\Delta L}{L}$$

$$\Rightarrow \epsilon_V = 2\epsilon_H + \epsilon_L$$

$$= 2 \frac{PD}{4tE}(2 - \mu) + \frac{PD}{4tE}(1 - 2\mu)$$

$$= \frac{PD}{4tE}(4 - 2\mu) + \frac{PD}{4tE}(1 - 2\mu)$$

$$= \frac{PD}{4tE}(4 - 2\mu + 1 - 2\mu)$$

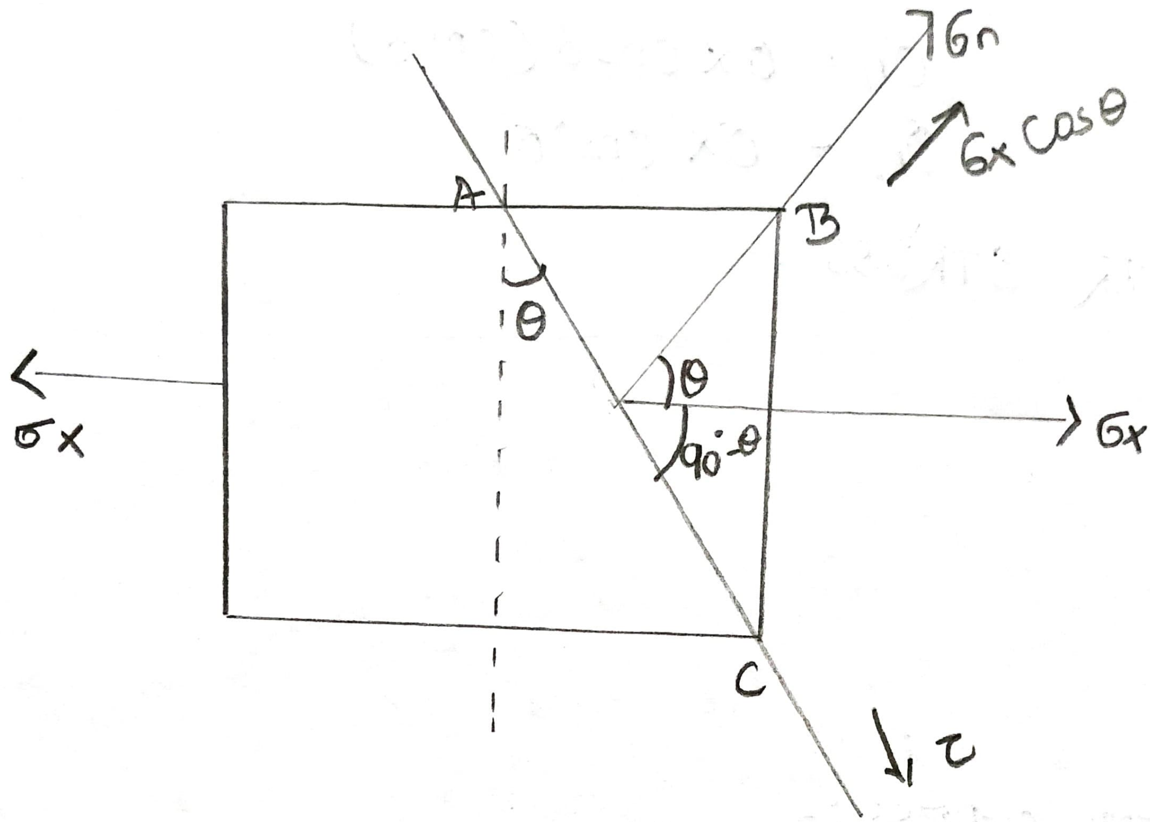
$$\Rightarrow \boxed{\epsilon_V = \frac{PD}{4tE}(5 - 4\mu)}$$

$$\frac{\Delta V}{V} = \frac{PD}{4tE}(5 - 4\mu)$$

* $\boxed{\Delta_V = \frac{PDV}{4tE}(5 - 4\mu)}$ 'ΔV' is change in volume.

TWO-DIMENSIONAL STRESS SYSTEM

CASE-1 WHEN MEMBER IS SUBJECTED TO UNIAXIAL DIRECT STRESS



NORMAL STRESS :-

$\Sigma F \perp^{r}$ AC PLANE $= 0$

$= \sigma_n (AC \times 1) = \sigma_x \cos\theta \, (BC \times 1)$

$= \sigma_n = \sigma_x \cos\theta \left(\frac{BC}{AC}\right)$

NORMAL STRESS :-

Σ FORCES $\perp^{r}$ AC PLANE $= 0$

$$\sigma_n (AC \times 1) = \sigma_x \cos\theta (BC \times 1) + \sigma_y \sin\theta (AB \times 1)$$

$$\sigma_n = \sigma_x \cos\theta \left(\frac{BC}{AC}\right) + \sigma_y \sin\theta \left(\frac{AB}{AC}\right)$$

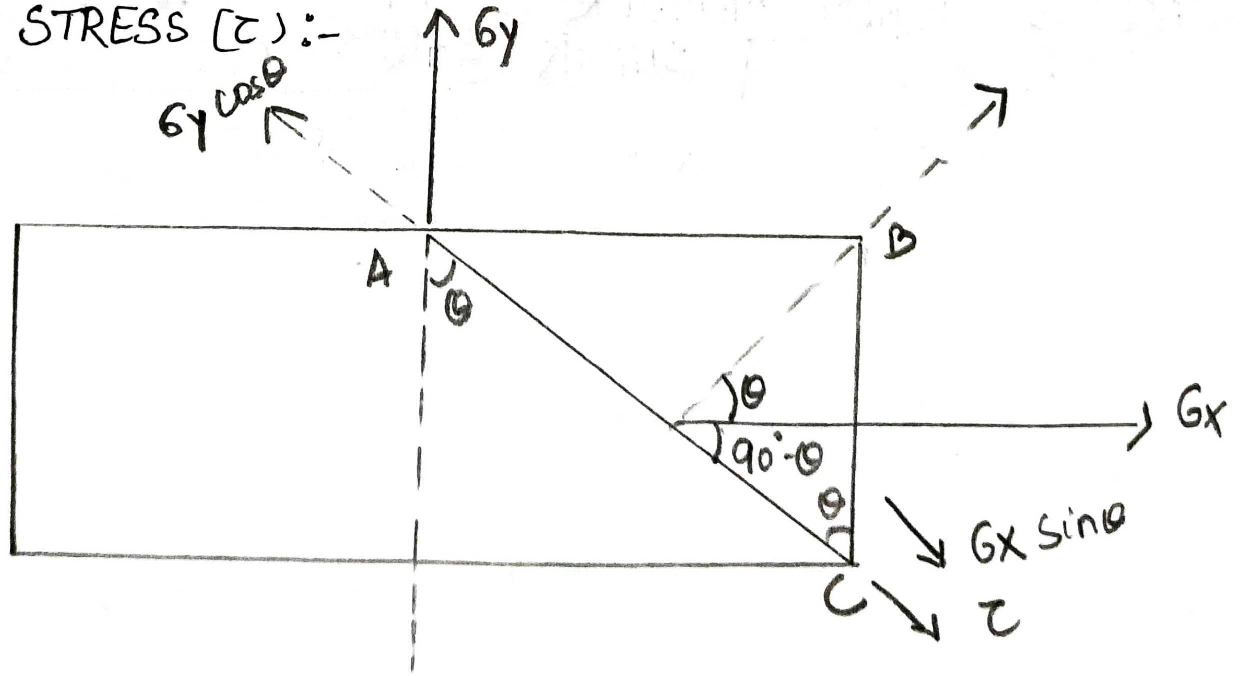
$$= \sigma_x \cos^2\theta + \sigma_y \sin^2\theta$$

$$= \frac{\sigma_x}{2}(1 + \cos 2\theta) + \frac{\sigma_y}{2}(1 - \cos 2\theta)$$

$$= \frac{\sigma_x}{2} + \frac{\sigma_x}{2}\cos 2\theta + \frac{\sigma_y}{2} - \frac{\sigma_y}{2}\cos 2\theta$$

$$\boxed{\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta}$$

SHEAR STRESS (τ) :-



RESULTANT STRESS (σ_R) :-

THE RESULTANT STRESS ON AN OBLIQUE PLANE CAN BE FOUND OUT BY

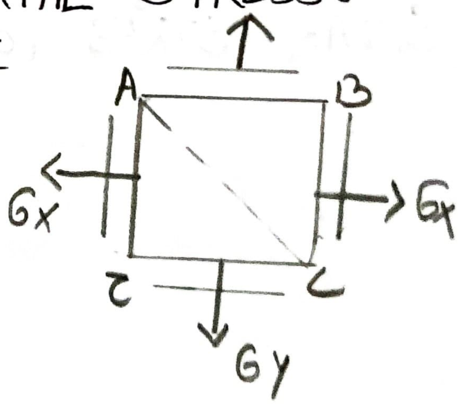
$$\sigma_R = \sqrt{\sigma n^2 + \tau^2}$$

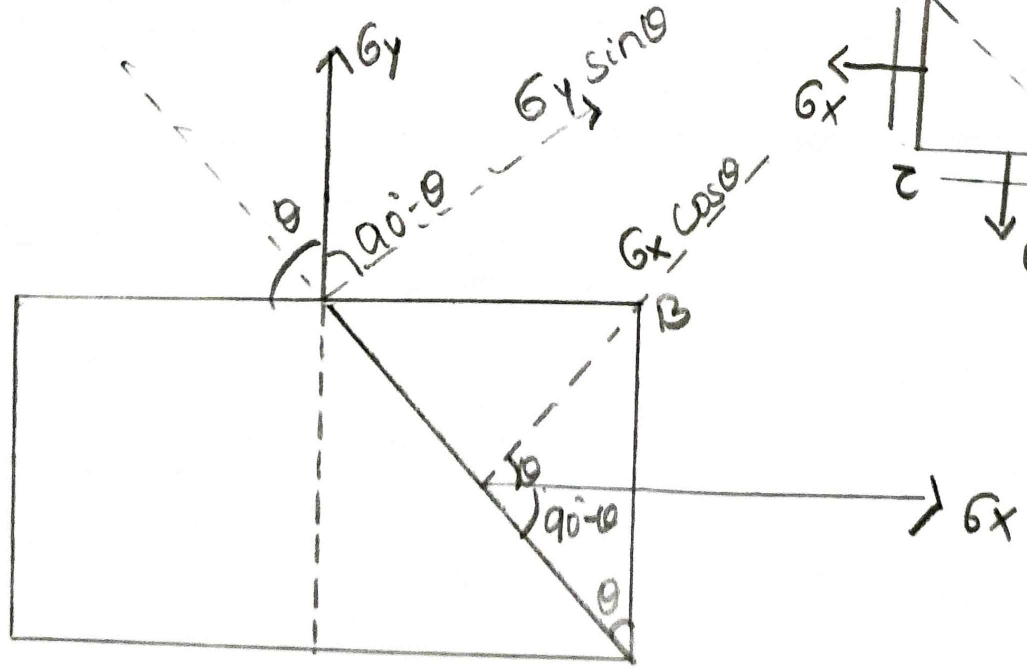
$$= \sqrt{(\sigma x \cos^2\theta)^2 + \left(\frac{\sigma x}{2} \sin 2\theta\right)^2}$$

$$\boxed{\sigma_R = \sqrt{\sigma n^2 + \tau^2}}$$

THIS IS THE EXPRESSION FOR NORMAL, STRESS SHEAR STRESS & RESULTANT STRESS DEVELOPED IN AN OBLIQUE PLANE AC WHEN S.T UNIAXIAL STRESS

CASE - 3 WHEN MEMBER IS S.T BIAXIAL STRESS :- & SHEAR STRESS τ





SHEAR STRESS :-

Σ FORCES $\parallel^{r}$ to AC PLANE $= 0$

$$\tau\,(AC \times 1) = -\sigma_x \sin\theta\,(BC \times 1) + \sigma_y \cos\theta\,(AB \times 1)$$

$$= -\sigma_x \sin\theta \left(\frac{BC}{AC}\right) + \sigma_y \cos\theta \left(\frac{AB}{AC}\right)$$

$$\therefore \tau = -\sigma_x \sin\theta \cos\theta + \sigma_y \cos\theta \sin\theta$$

$$= -\frac{\sigma_x}{2} \sin 2\theta + \frac{\sigma_y}{2} \sin 2\theta$$

$$\tau = -\frac{1}{2}(\sigma_x - \sigma_y) \sin 2\theta$$

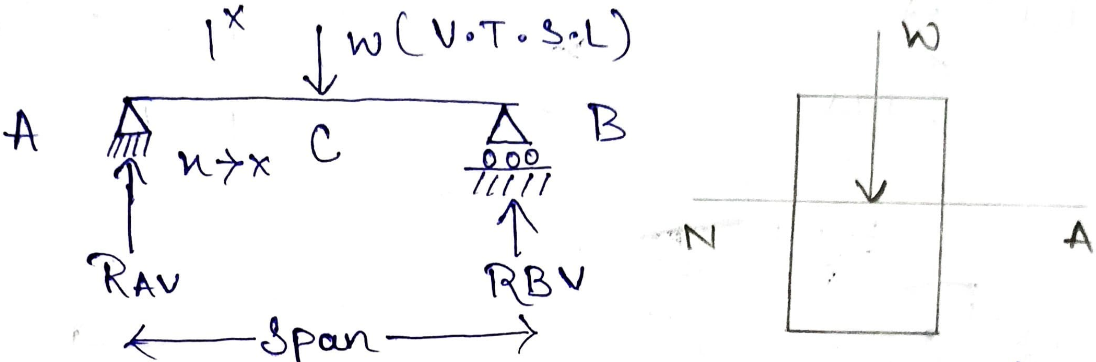
$$\boxed{\therefore \tau = -\frac{1}{2}(\sigma_x - \sigma_y) \sin 2\theta}$$

$$\boxed{\because \sigma_R = \sqrt{\sigma_n^2 + \tau^2}}$$

SHEAR FORCE / BENDING MOMENT

- A beam is defined as any structural member which is subjected to transverse shear load (TSL).
- Because of TSL, Beams are subjected to variable shear load & variable B.M.

Ex.-



X-Section of Beam of C

- It is represented by its
  - Longitudinal Axis
  - Support Reaction
  - Loads acting on it
  - Span

To know variable B.M & S.F and their moxm value ; SFD & BMD are to be drom. SFD & BMD are useful in the design of become & shaft bared and Therie of Failure on strength & rigidity criteria.

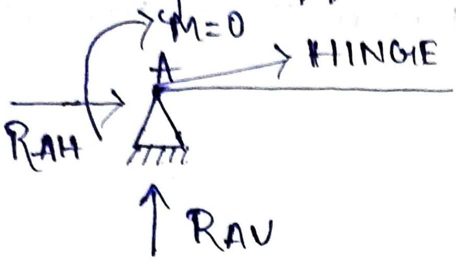
TYPES OF SUPPORTS :-

2- Simple Supports

- (b) → Roller support (RH = 0)
  So It Restricts Horizontal Motio
- → Hinge support / Pin support

It restricts horizontal & vertical motion.

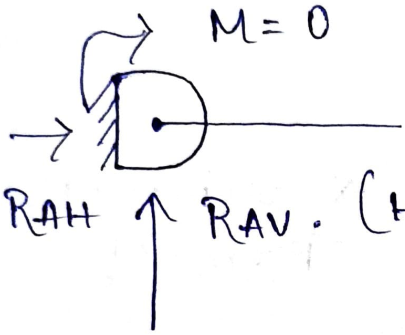
Hinge Support Representation :-



$(AL)_A = -R_{AH}$

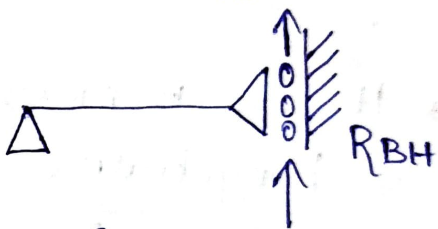
$(SF)_A = R_{AV}$

$(BM)_A = 0$



(Hinge Support)

Bcoz motion is Possible



Roller support → only 1st has 1 Reaction. $R_{BV}=0$
It can have horizontal / vertical reaction depending on the position of roller support.

TYPES OF BEAMS :-

BEAMS

- STATICALLY DETERMINATE BEAMS. (TOTAL NO. OF REACTION IN A BEAM) = NO. OF USEFUL EQn).
- STATICALLY INTERMINATE BEAM [TOTAL NO. OF REACTION IN A BEAM > NO. OF USEFUL ST. EQn]

Simply Supported Beam

Overhanging Beam

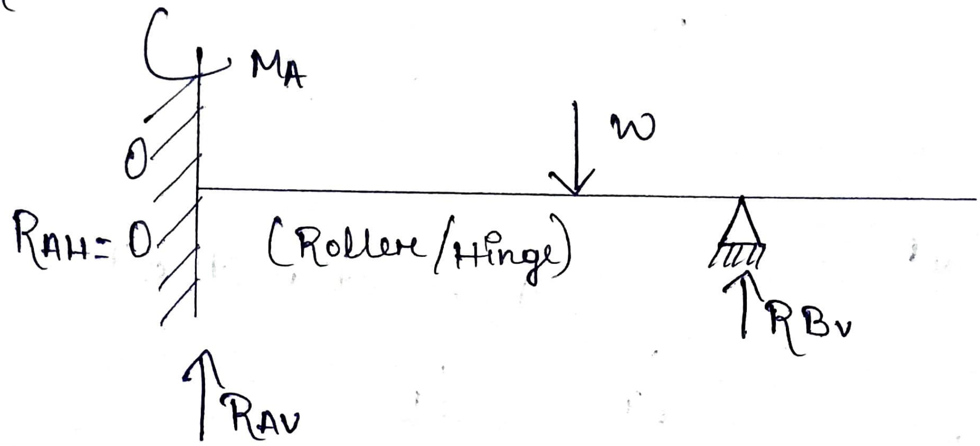
Cantilever Beam

Fixed or Built in Beam

Propped cantilever Beam

Continous Beam

Ex:-

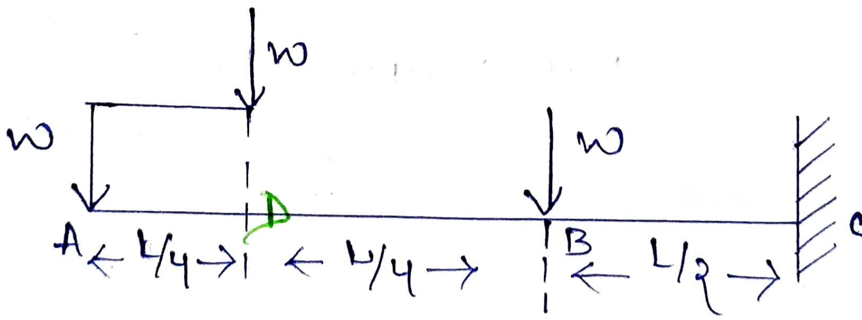


No. of Reaction = 3

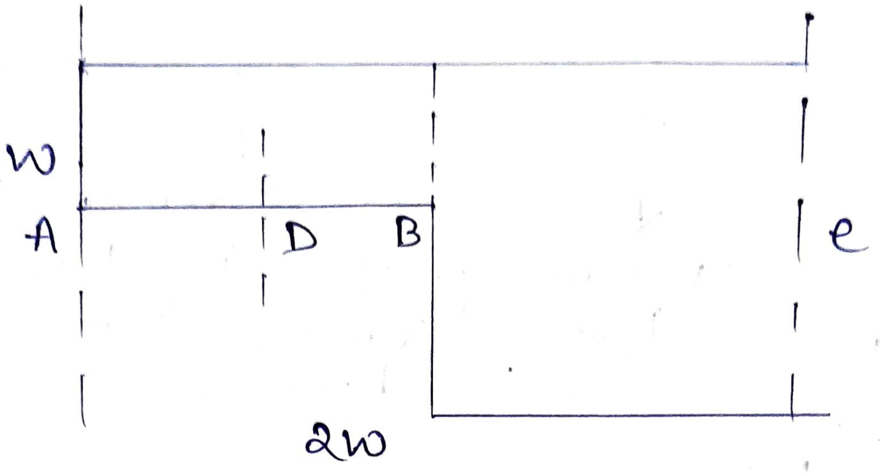
No. of useful static Eqⁿ = 2

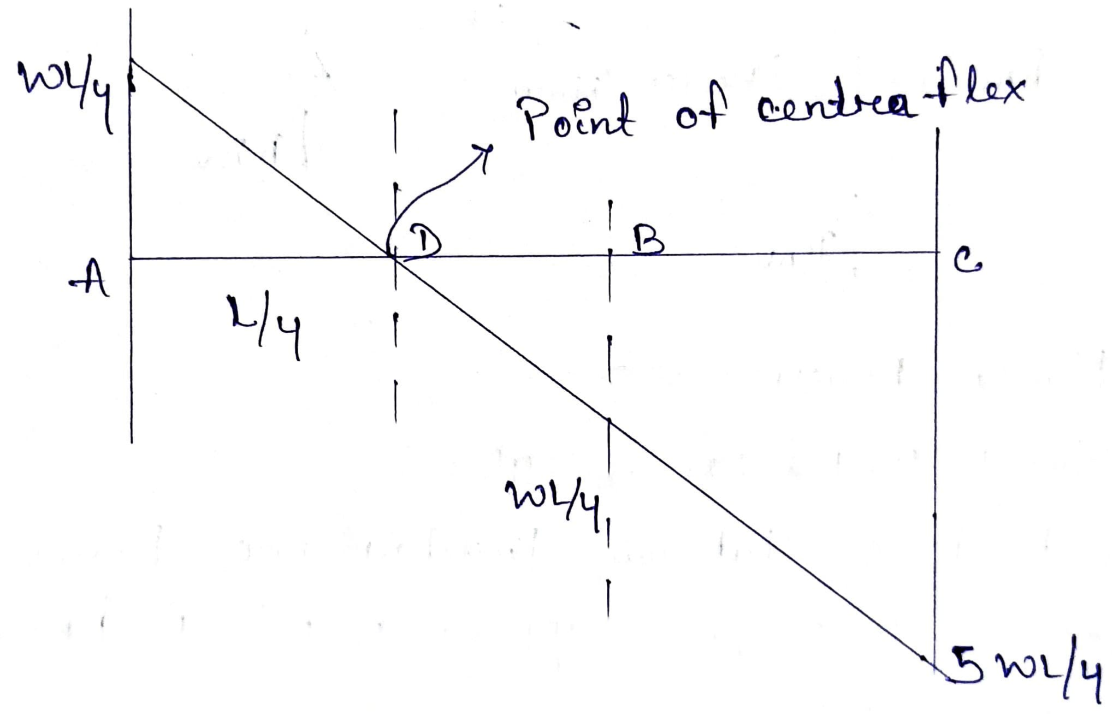
So It is a Statically Indeterminate Beam

Prop is a simple support which is Provided near free end.



Draw SFD, BMD, Max^m Sagging & Hogging B.M.





In a cantilever beam, vertical reaction at the fixed End = - (Net vertical Load)

Horizontal Reaction of fixed End = - (Net Axial Load)

Moment reaction of fixed end = (Net Moment at fixed End)

Net moment / c $= w\left(\frac{L}{2}\right) - wL + wL/4 = \frac{5wL}{4}$

Shear Force :-

$(SF)_A = (SF)_D = (SF)_B = -w$

$(SF)_B = -2w = (SF)_c$

Bending Moment :-

$(BM)_A = wL/4$

$(BM)_B = \frac{wL}{4} - \frac{wL}{2} = \frac{-wL}{4}$

$(BM)_c = \frac{wL}{4} - wL = -5wL/4$

$(BM)_c = -5wL/4 + 5wL/4 = 0$

CLASSIFICATION OF BEAMS :-

BEAM

- Statically DETERMINATE BEAM (No. of zero Reaction = No. of USEFUL STATIC Equili Eqn)
  - SIMPLY SUPPORTED BEAM
  - OVERHANGING BEAM
  - CANTILOVER BEAM

  SIMPLY SUPPORTED BEAM and OVERHANGING BEAM → STAT. DETERMINATE BEAMS EXCEPT WHEN THEY ARE SUPPORTED BY 2 HINGE SUPPORTS IN PRESENCE OF AXIAL LEAD

- STATICALLY ~~INTER+~~ INDETERMINATE BEAM
  REACTION ARE DETERMINED WITH THE HELP OF STATIC Equilibrium Eqn & COMPATIBILITY
  - FIXED OR BUILT-IN BEAM
  - PREPPED CANTILOVER BEAM
  - CONTINIOUS BEAM

S.S.B :-

It is a beam which is supported at its extreme ends by 2 simple supports i.e. at 2 roller support or 2 hinges or 1 hinge & 1 roller.

OVERHANGING BEAM:-

At least one support is provided away from the support. It is supported by 2 supports.

CANTILOVER BEAM:-

→ Only 1 support reqd.
→ It is always statically determinate beam.

FIXED/BUILT-IN BEAM:-

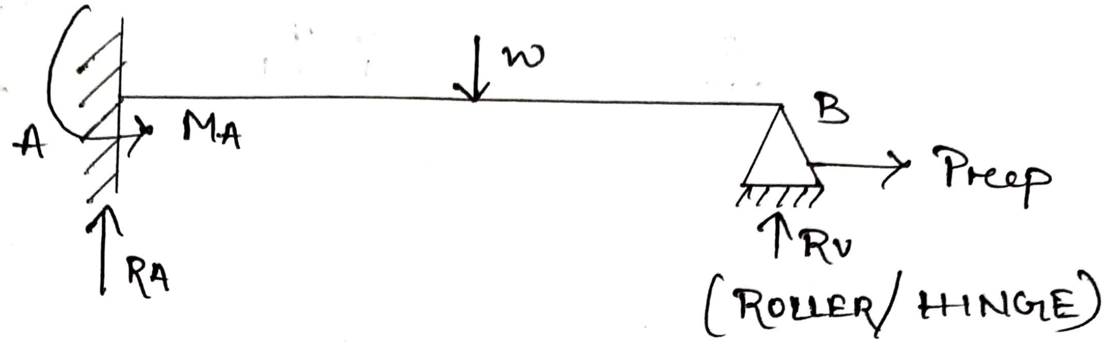
→ It is a beam which is always supported by 2 fixed supports.

→ It is always statically Indeterminate Beam.

PROPPED CANTILEVER BEAM:-

→ Remaining either simple supports or fixed support. But it is a combination of supports, i.e we can say it is a modified cantilever Beam, where one extreme end is having fixed support of near other end is having simple support i.e (roller/Hinge)

Prop → A simple support having fixed support of one end is having simple support (Hinge/roller support)



3 unknown Reaction & 2 equations

$\Sigma F = 0$ & $\Sigma M = 0$

So It is a Statically Indeterminate Beam

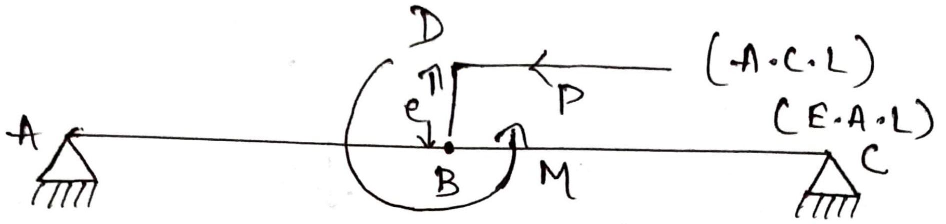
No. of non-zero Reaction > No. of Equations

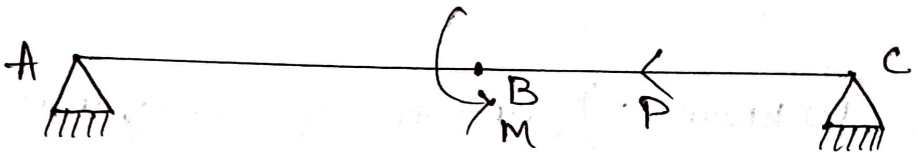
CONTINIOUS BEAMS :-

- It is a modified simply supported Beam. Minimum no. of supports required in other beam = 2. But here minimum no. of supports required = 3 that can be 4 or 5 also.
- So continous beam is a beam which is not only supported at its extreme ends but also supported by its Intermediate Points.

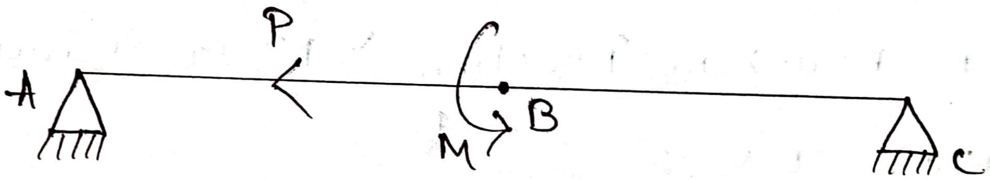
It is supported by simple supports :-

(Statically Indeterminate Beam)





So



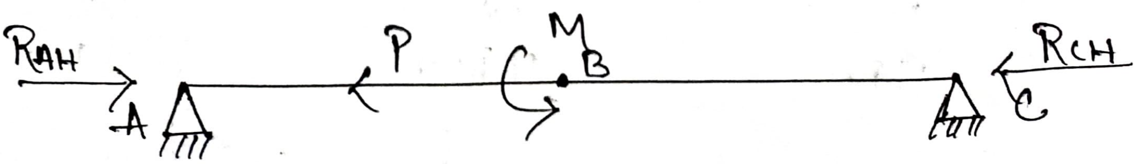
S.S.B $\rightarrow$ Statically Indeterminate Beam

$\rightarrow$ No. of non-zero Reaction = 4
i.e R_{AH}, R_{AV}, R_{CV} ; R_{CH}

$\rightarrow$ Useful Equilibrium Eqn = 3
$\Sigma V = 0$, $\Sigma H = 0$ & $\Sigma M = 0$

$\rightarrow$ Compatibility Eqn = 1
$(\delta_L)_{AC} = (\delta_L)_{AD} + (\delta_L)_{BC} = 0$

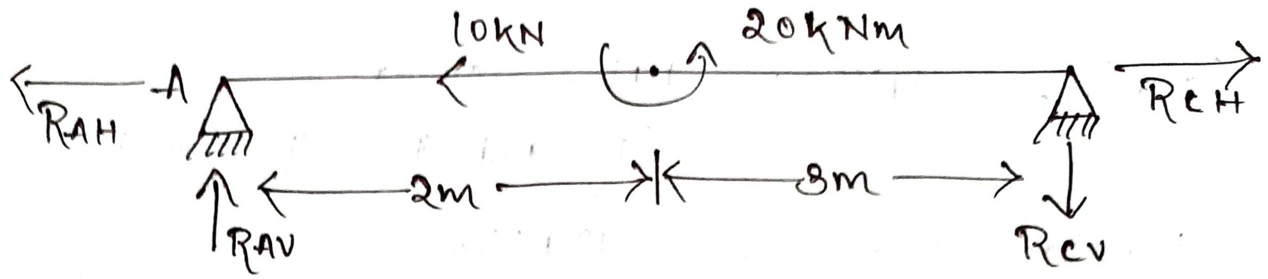
EX :-



$R_{AH} = \dfrac{P(BC)}{AC}$ $(\longrightarrow)$

$R_{CH} = \dfrac{P(BA)}{AC}$ $(\longrightarrow)$

Q)



$\therefore R_{AH} = \frac{(10)3}{5} = 6KN$

<u>Soln</u> :- $R_{CH} \; \frac{(10)2}{5} = 4KN$

Here vertical Reactions = 0

$\therefore R_{AV} + R_{CV} = 0$

So, they are equal & opposite

So, $\Sigma M_A = 0$

$\Rightarrow R_{CV} \times 5 + 20 = 0$

$\Rightarrow R_{CV} = -4KN$

$R_{AV} = 4KN$

So By Taking moment by A = 0

It is Proved that; These reactions are equal and opposite.

So ALA ; AL = − 6KN — ($\rightarrow$ -ve)

SF = 4KN ; BM = TM = 0

ALC ; AL = 4KN

SF = 4KN ; BM = TM = 0

ALB ; $(AL)_{AB}$ = − 6KN (only consider left section)

B belongs to AB Portion (By Neglecting applied A.L at B)

$(AL)_{BC} = -6 + 10KN$

$= 4KN$

$\therefore (AL)_B = -6 kN$ (Larger value)

$(B.M)_{AB} = 4 \times 2 = 8 \ kNm$ (cw +ve)

(By neglecting B.C at B)

$(B.M)_{BC} = 8 - 20 = -12 \ kNM$

(By considering BC at B)

$(BM)_D = -12 \ kNm$. (Maximum value)

As there is no concentrated point load acting on the beam. So there are only 1 S.F calculation i.e from left of the section which remains same throughout the beam.

$(S.F)_B = 4 kN (\uparrow) = +ve$

When you have to calculate A.L, S.F & B.M at the junction, then take 2 sections & calculate the applied loads on the extreme left section by neglecting applied load on the junction as shown above. Then maximum value is taken as the applied A.L, S.F & BM.

Resistance To Shear Force Is Called B.M.

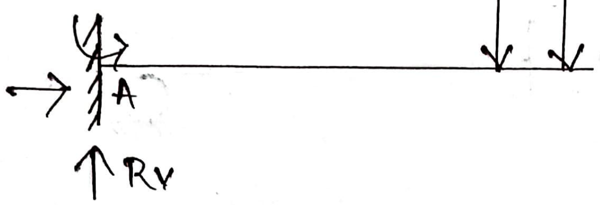
Types of Supports :-

1. Fixed Support / Built-In / Clamped Support :-

* No. of Reaction at any Support = No. of Restricted Motion by that Support

* Resistance Against Rotation — B.M

* Resistance Against Displacement / Motion — Shear Force / Reaction Force



$(AL)_A = R_H$

$(SF)_A = R_X$

$(BM)_A = -M$

2. Simple Supports :-

a) Roller Support :-

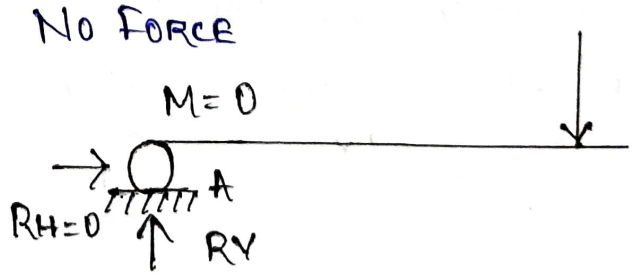
Rolling In Horizontal Direction
Hence No Resistance Against Horizontal Motion. So $R_H = 0$

As No Resistance = No Force

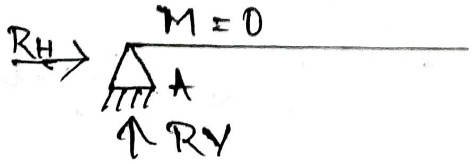
$(AL)_A = 0$

$(SF)_A = R_{AV}$

$(BM)_A = (\uparrow M)_A = 0$



b) Hinge Support (Fixed At A Point)

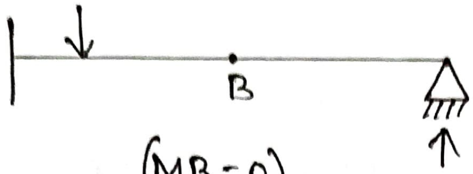


$(AL)_A = -R_{AH}$

$(SF)_A = R_{AV}$

$(BM)_A = 0$

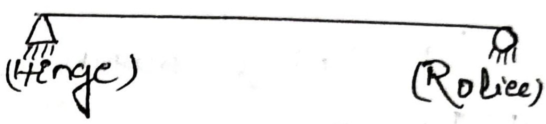
3. INTERNAL HINGE :-

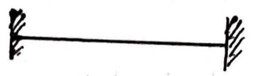
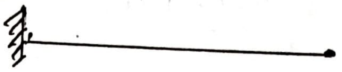


($M_B = 0$)

TYPES OF BEAMS :-

Beams are mainly of 4 type

1. Simple Supported Beam 

2. Fixed Beam 
3. Cantilover 

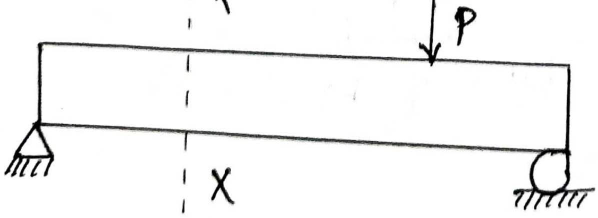
4. ~~Sup~~ Propped cantilover 

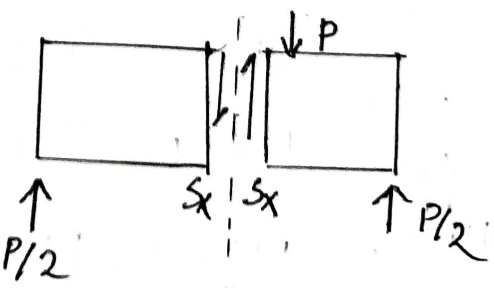
SHEAR FORCE :-

It is the internal resisting transverce force that require to convert a free body diagram in to equilibrium either from the left of the section or from the right of the section

Or

It can be defined as the summation of all the transverse force either from the left or right of the section





$S_x = \frac{P}{2}$ (In Equilibrium)

SIGN CONVENTION :-

+ve S.F

-ve S.F

BENOING MOMENT :-

It is the resultant moment at any section either from the left / right of the section

Coupled about longitudinal axis - Torque

" Transverse axis - Moment

M_1 = Torque (Twisting / Rotation occurs)

M_2 & M_3 = Bending moment

(Bcez of their Bonding)

-> If moment M_3 only acts about Z-axis, then Z is called Netural axis. The axis about which the body will be under pure rotation is called Neutral axis.

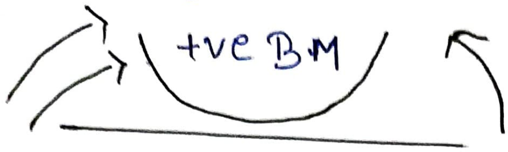
-> After Bonding, Top fibree will get comprosued & Bottom fibre gets expanded & vice-versa

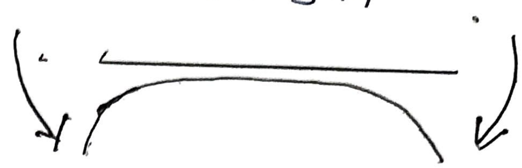
-> If C/s of the beam is rectangular, then it become Trapizoidal after bonding.

-> Max^m stress is developed alongs against deformation

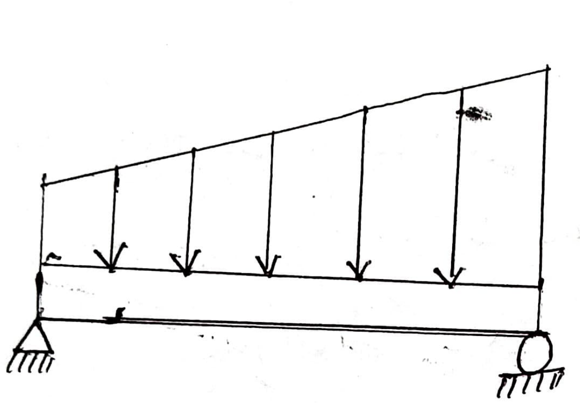
$$\boxed{\frac{\sigma}{y} \quad \frac{M}{I} = \frac{E}{R}}$$

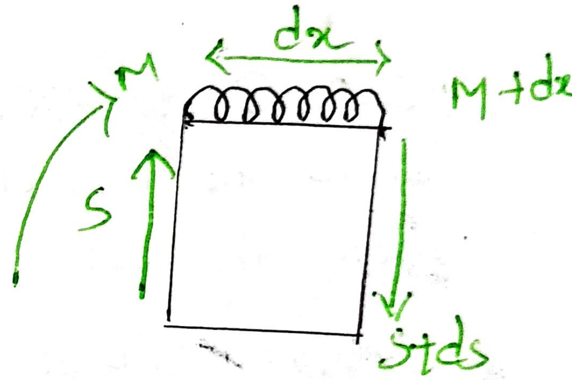
SIGN CONVENTION





REACTION B/W LOADING RATE, SHEAR FORCE & BENDING MOMENT:-





Assumption:-

let us assume that the shear force & B.M are +ve on the elemental section.

Force equilibrium in vertical direction

$$S-(S+ds)-Wdx=0$$

$$\Rightarrow -ds = wdv$$

$$\boxed{-\left(\frac{ds}{dx}\right) = w} \rightarrow \text{LOADING RATE}$$

For point Load; Slope = 0

It means that, -ve slope of shear force curve at any section is equal to the loading rat at that section

Couple about 1-1

$$M-(M+dm)+(S+ds)dx+(wdx)\frac{dx}{2}=0$$

$$\Rightarrow -dm+(S+ds)dx+\frac{wdv^2}{2}=0$$ Moment CW +ve, CCW -ve

$$\Rightarrow -dM+Sdx=0$$

$$\Rightarrow dM=Sdx$$

$$=\boxed{\frac{dM}{dx}=S}$$

This expression shows that the +ve slope of the bending moment curve of any section is equal to the magnitude of shear force at that section

Note :-

1- If the loading is concentrated (point load). SFD will be rectangular & BMD will be triangular.

2- If loading is UDL. SFD will be triangular & BMD will be parabolic (2nd order)

3- If the loading is UVL, then SFD will be parabolic & BMD will be cubic.

4- If shear force changes its sign at a section BM will be maximum at that section.

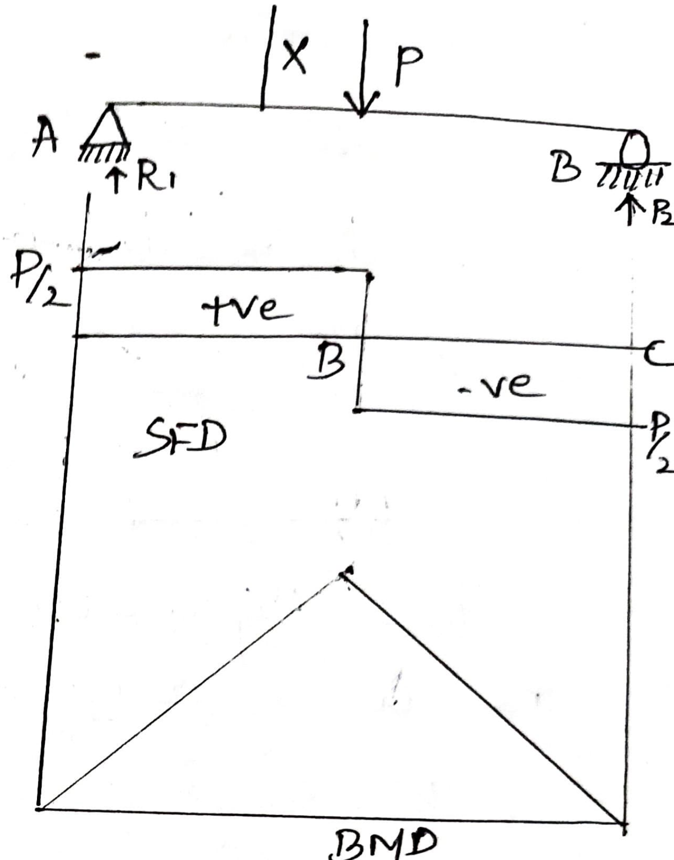
5- IF B.M changes its sign at a point. The curvature also changes its sign at that point such point is known as point of contraflexture or point of inflextion

Q.1) $R_1 + R_2 = P$

$\Sigma M_A = 0$

$\Rightarrow R_2 \times L - P \times \frac{L}{2} = 0$

$\Rightarrow R_2 = \frac{P}{2}, \; R_1 = \frac{P}{2}$



SFD

At Section AB; $(AL)_{xx} = 0$

$S_x = +R_1 = \frac{P}{2}$ $(SF)_{xx} = \frac{P}{2}$

So $S_A = S_B = \frac{P}{2}$

At Section BC :-

$S_x = +R_1 - 1 = \frac{P}{2} - P = \frac{-P}{2}$ So $S_B = S_C = \frac{P}{2}$

BMD:-

M_x (Between A & B) $= +R_1 x = \frac{P}{2} x$

At $x, 0$; $M_A = 0$

$x = \frac{L}{2} = M_B = \frac{PL}{4}$

$M_x (B \& C) = R_1 (x + \frac{L}{2}) - Px$

$= \frac{P}{2} (x + \frac{L}{2}) - Px$

$M_C = PL/4$ $M_D = 0$

2. UDL

$R_A + R_B = WL$ ...(1)

$\Sigma M_A = 0$

$\Rightarrow R_B \times L = (WL) \frac{L}{2}$

$\Rightarrow R_B = \frac{WL}{2}; \; R_A = \frac{WL}{2}$

SFD :-

$S_x = +R_A - Wx$

$= \frac{WL}{2} - Wx$

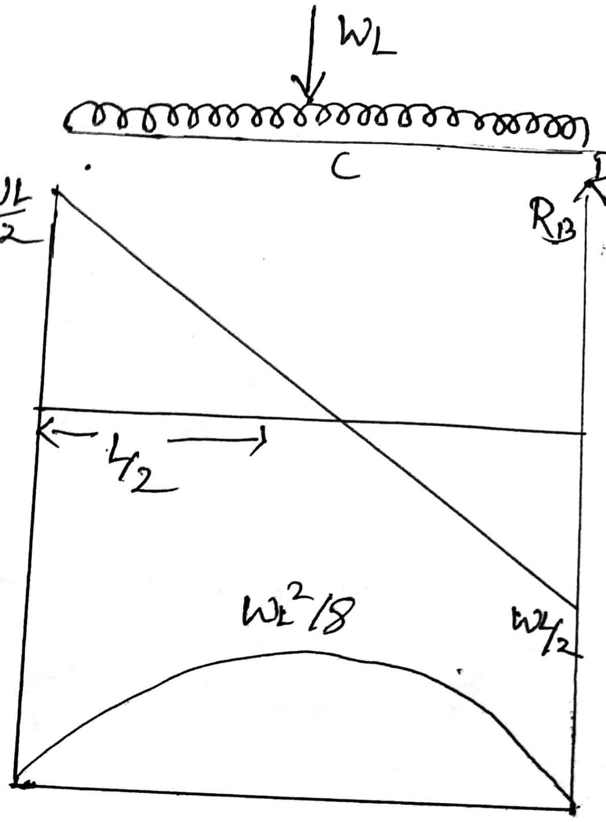
At $x = 0;\ S_A = \frac{WL}{2}$

$x = \frac{L}{2};\ S_C = 0$

$x = L;\ S_B = \frac{WL}{2} - WL = -\frac{WL}{2}$

$S_x = 0$

$\Rightarrow \boxed{x = \frac{L}{2}} \rightarrow$ It change sign



BMD :-

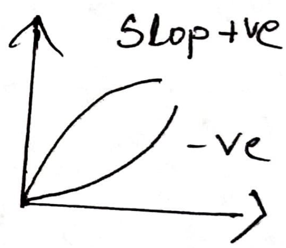
$M = R_1 x - \frac{Wx^2}{2} = \frac{WL}{2}x - \frac{Wx^2}{2}$

At $x = 0;\ M_A = 0$

$x = \frac{L}{2}\ M_C = \frac{WL^2}{4} - \frac{WL^2}{8} = \frac{WL^2}{8}$

$x = L;\ M_B = \frac{WL^2}{2} = \frac{M^2}{2} = 0$

$\frac{dM}{dx} = S(+ve)\ ;\ \left(\frac{dM}{dx}\right)_A\ S_A = \frac{WL}{2}$ +ve



3-UVL :-

$R_A + R_B = \frac{1}{2} \times 5 \times 5$

$= 12.5\ KN$

$\Sigma M_A = 0$

$\Rightarrow R_B \times 5 - \left(\frac{25}{2}\right) \times \frac{2}{3} \times 5 = 0$

$\Rightarrow R_B = \frac{25 \times \cancel{5}}{3 \times \cancel{5}} = \frac{25}{3}\ KN$

S_x (in AB) (x from A)

$S_x = +R_1 - \frac{1}{2} x x$

$= \frac{25}{6} - \frac{1}{2} x^2$

At $x = 0$; $S_A = \frac{25}{6}$

$x = 5$; $S_B = \frac{25}{6} - \frac{25}{2} = 25\left(\frac{1-3}{6}\right)$

$= -\frac{25}{3}$ M

As It change Sign

So. $S_x = 0 \Rightarrow \frac{25}{6} = \frac{x^2}{2} \Rightarrow x^2 = \frac{50}{6}$

$x = \frac{\sqrt{50}}{\sqrt{6}}$

B.M in AB (in from A)

$M_n = +R_1 x = \frac{x^2}{2}\left(\frac{x}{3}\right)$

$\Rightarrow \frac{25}{6} x - \frac{x^3}{6}$

At $x = 0$; $M_A = 0$

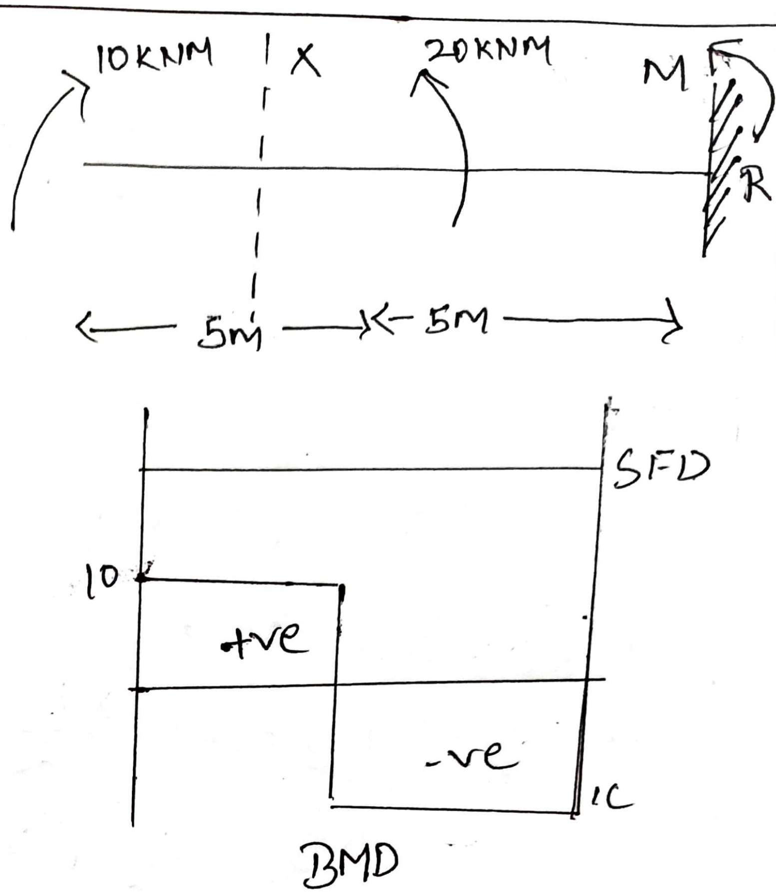
$x = 5$; $M_B = 0$

$x = \frac{5}{\sqrt{3}}$, $M_C = 8.01$

→ If a pure conc. load is applied at a point on the beam, then there will be a sudden change in sheare force at that point with the same magnitude as that of point Load.

→ Similarly if a pure moment is applied over a beam at any point, then there will be sudden change in B.M at that point with magnitude of that moment.

4.



As $R=0$

$S_x = 0$

So $S_A = S_B = S_C = 0$

(Bet $A \& B$: S.F $= 0$

$B \& C$; S.F $= 0$)

<u>BMD</u>

$M_x = +10$

So $M_A = M_B = 10$ KNM

B.M in $B \& C$;

$M_x = 10 - 20 = -10$ KNM

So $M_B = M_C = -10$ KNM

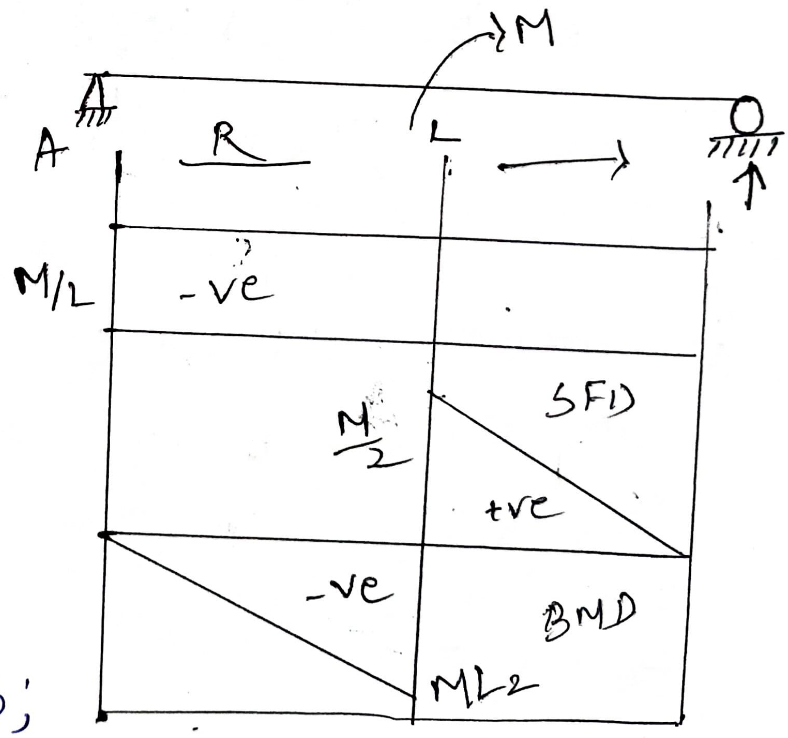
5) So $R_1 + R_2 = 0$

$\Sigma M_A = 0$

$\Rightarrow R_2 \times L = M$

$\Rightarrow R_2 = \frac{M}{L}$

$R_1 = -\frac{M}{L}$



Sheare force B/w $A \& B$;

$S_x(AB) = S_x = +R_1 = -\frac{M}{L}$

$S_A = S_B = -\frac{M}{L}$ (as there are no other force acting on the beam)

Similarly Between B & C;

$S_x(BC) = +R_1 = -\frac{M}{L}$ (Remains constant throughout)

$S_B = S_C = -\frac{M}{L}$

BMD:

$M_x = +R_1 x$ (x varies from A to B)

$M_A (x=0)$; $M_A = 0$

$x = \frac{L}{2}$; $M_B = +R_1 \times \frac{L}{2} = -\frac{M}{L} \times \frac{L}{2} = -\frac{M}{2}$

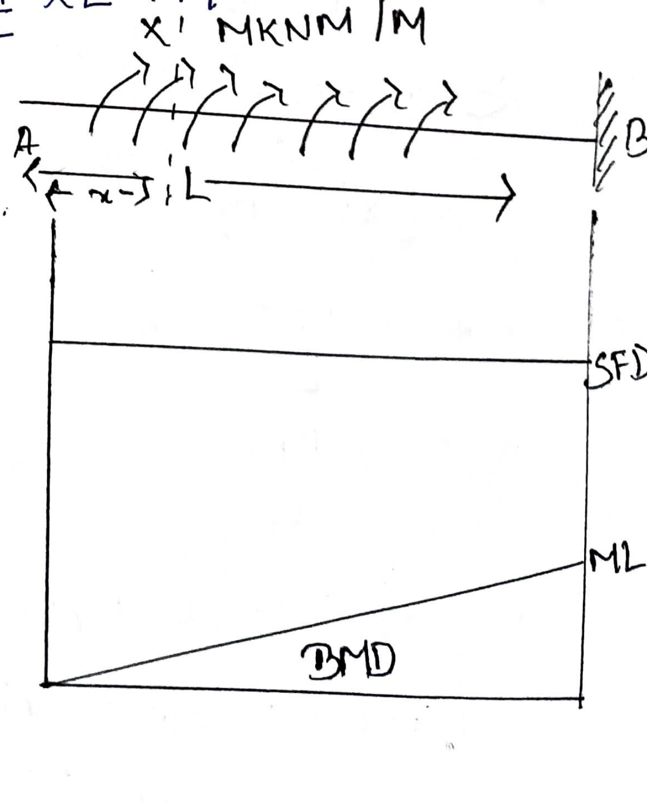
$M_B = -\frac{M}{2}$ (-ve) from A to B

As SFD is $\square$ BMD is Δ

Similarly B in B & C; $M_x = +R_1 (x + \frac{L}{2}) + M$

So At B: $M_B (x=0) = -\frac{M}{L} (\frac{L}{2}) + M = \frac{M}{2}$ +v

$M_C (x = \frac{L}{2}) = -\frac{M}{L} \times L + M = 0$



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As there is no force acting on the beam

So $R = 0$

SFD

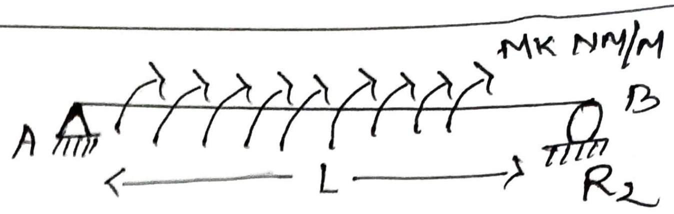
$S_A = S_B = 0$

BMD

$M_x = +M_x$

$M_A (x=0)$; $M_A = 0$

$M_B\ (x=L)\ ;\ M_B = ML$



7

$R_1 + R_2 = 0$

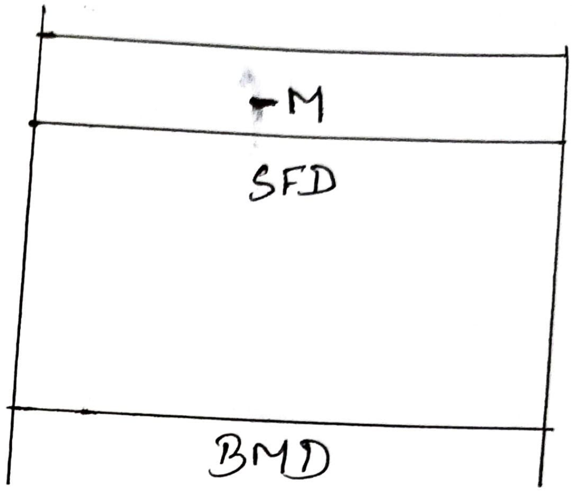
$\Sigma M_A = 0$

$\Rightarrow R_2 \times L = M \times L$

$\Rightarrow R_2 = M;\ R_1 = -M$

So <u>SFD</u>

$S_x = +R_1 = -M$



It remains constant throughout the beam

As there is no other force acting on the beam

So $S_A = S_B = -M$

<u>B.M.D</u>

$M = +R_1 x + Mx$

At $x=0;\ M_A = 0$

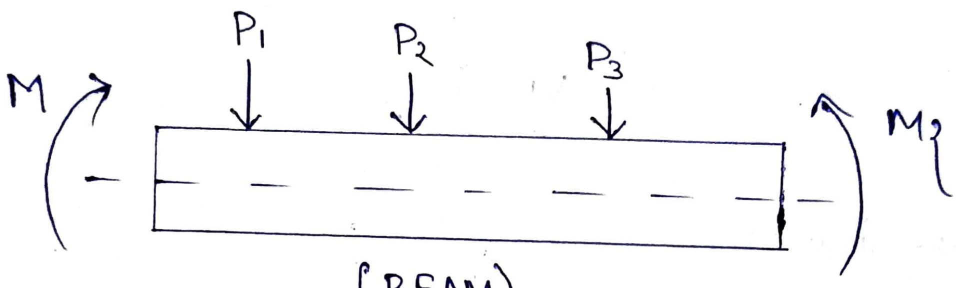
$x = L\ M_B + R_1 L + ML$

So $M_B = -ML + ML = 0$

So Between A & B ; B.M = 0 throughout the length of the beam. Hence BMD will be a straight line.

THEORY OF BENDING OF BEAMS

- MEMBERS SUBJECTED TO LOADS TRANSVERSE TO THE AXIS ARE TERMED AS BEAM
- THE MEMBERS ARE SUBJECTED TO FORCES/MOMENT HAVING THEIR VECTORS PERPENDICULAR TO THE AXIS OF THE BAR.

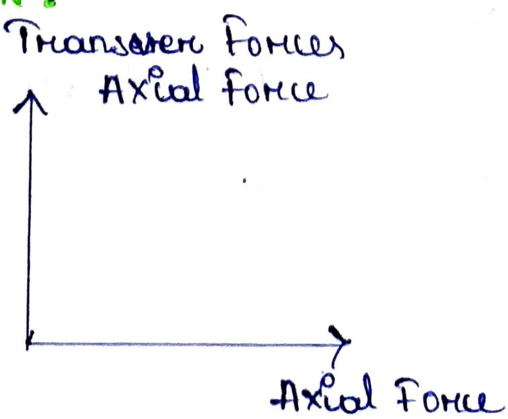


(BEAM)

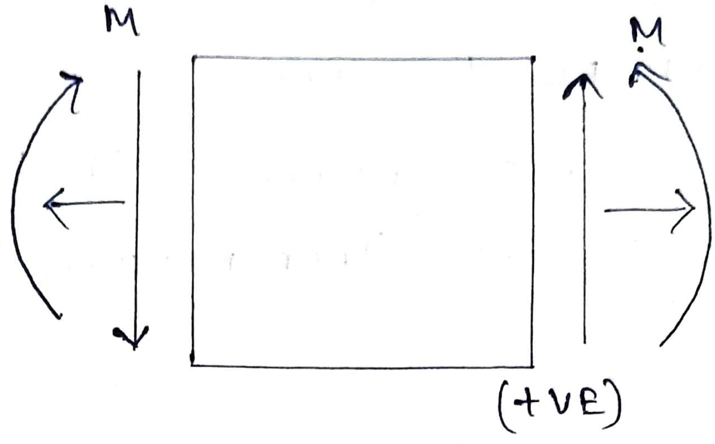
- These BEAMS Are Supported By
  - i) HINGE SUPPORT
  - ii) ROLLER SUPPORT
  - iii) FIXED SUPPORT

So whenever, THESE Transverse LOADS ACT ON THE BEAM, It will be Subjected to Bending.
This Is Accompained by Transverse SHEAR FORCE.

SIGN CONVENTION :-

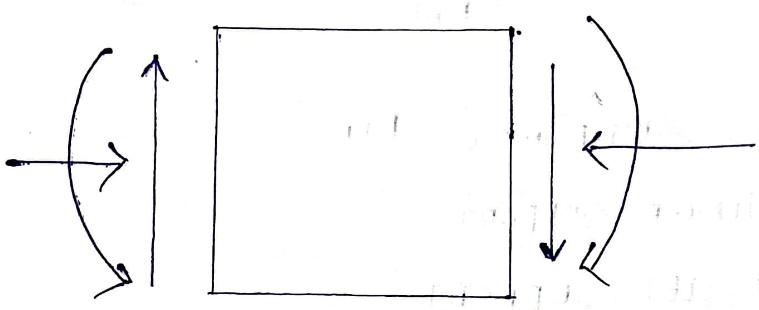


SAGGING MOMENT :-

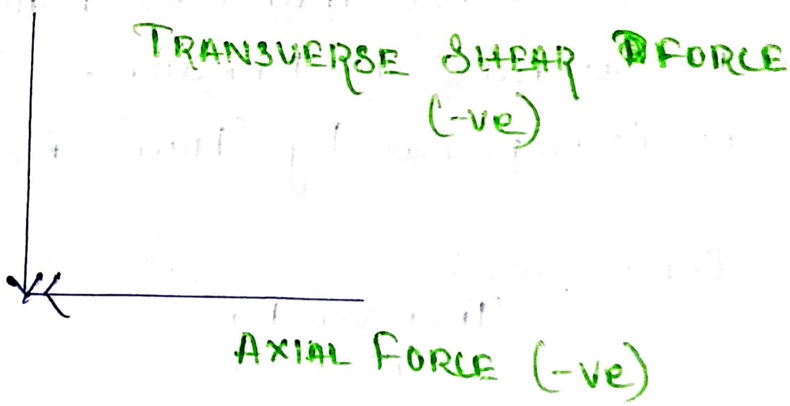


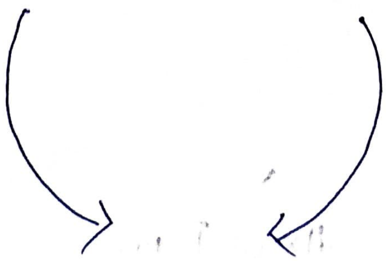
(+VE BM)

HOGGING MOMENT :-



(-VE moment)





(M is -ve)

BENDING EQUATION :-

$$\boxed{\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}}$$

ELIATZABETH, RANI MAY I FOLLOW
YOU

WHERE

E → YOUNG'S MODULUS OF ELASTICITY

R → RADIUS OF CURVATURE

M → MOMENT CARRYING CAPACITY

I → MOMENT OF INERTIA

F = σ → STRESS DEVELOPED

Y → DISTANCE OF TOP FIBRE FROM NEUTRAL AXIS

ASSUMPTION :-

i) MATERIAL IS HOMOGENEOUS, ISOTROPIC & ELASTIC

ii) YOUNG'S MODULUS OF ELASTICITY (E) REMAINS SAME IN TENSION AS WELL AS COMPRESSION.

iii) PLANE SECTION REMAINS PLANE AFTER BENDING.

iv) STRESSES ARE WITH IN ELASTIC LIMIT.

v) BEAM IS INITIALLY STRAIGHT & EVERY LATER OF IT IS FREE TO EXPAND.

MOMENT OF INERTIA :-

MOMENT OF INERTIA IS DEFINED AS THE QUANTITY EXPRESSED BY THE BODY RESISTING ANGULAR ACCELERATIO[N] WHICH IS THE PRODUCT OF MASS & OF EVERY PARTICLE WITH ITS SQUARE OF DISTANCE FROM THE AXIS OF ROTATION.

OR

It CAN BE DESCRIBED AS A QUANTITY THAT DECIDES THE AMOUNT OF TORQUE NEEDED FOR A SPECIFIC ANGULAR ACCELERATION IN A ROTATIONAL AXIS

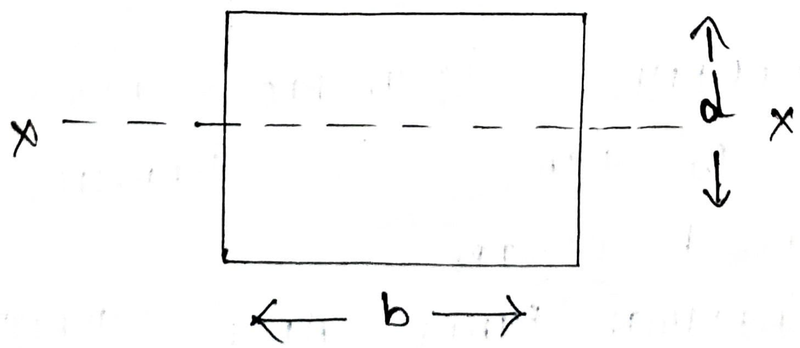
SO $I = MK^2$ OR $T = I\alpha$

$$I = \cancel{MK^2}\, mk^2 \ (kg\,m^2)$$

OR

$$T = I\alpha \Rightarrow I = \frac{T}{\alpha}$$

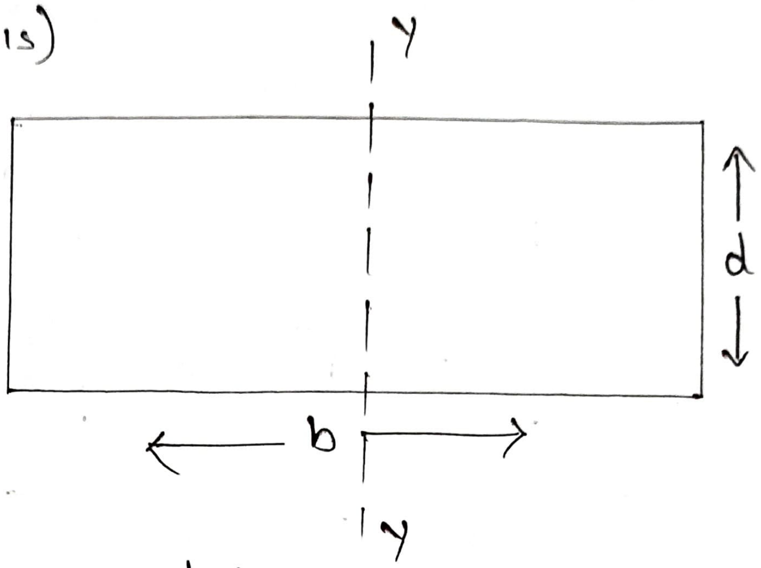
RECTANGULAR SECTION :-



$$\therefore I_{xx} = \frac{bd^3}{12}$$

RECTANGULAR SECTION :-

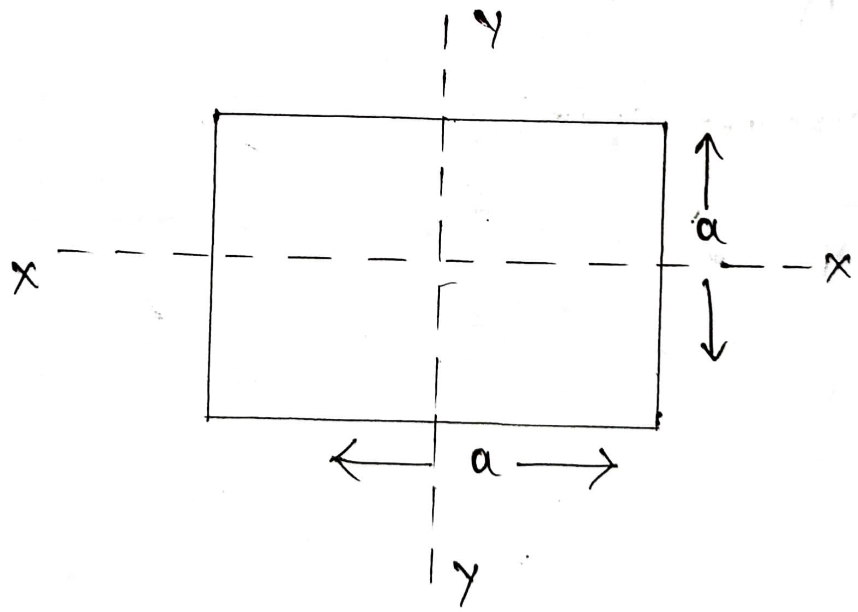
(ABOUT YY AXIS)



$$\therefore I_{yy} = \frac{db^3}{12}$$

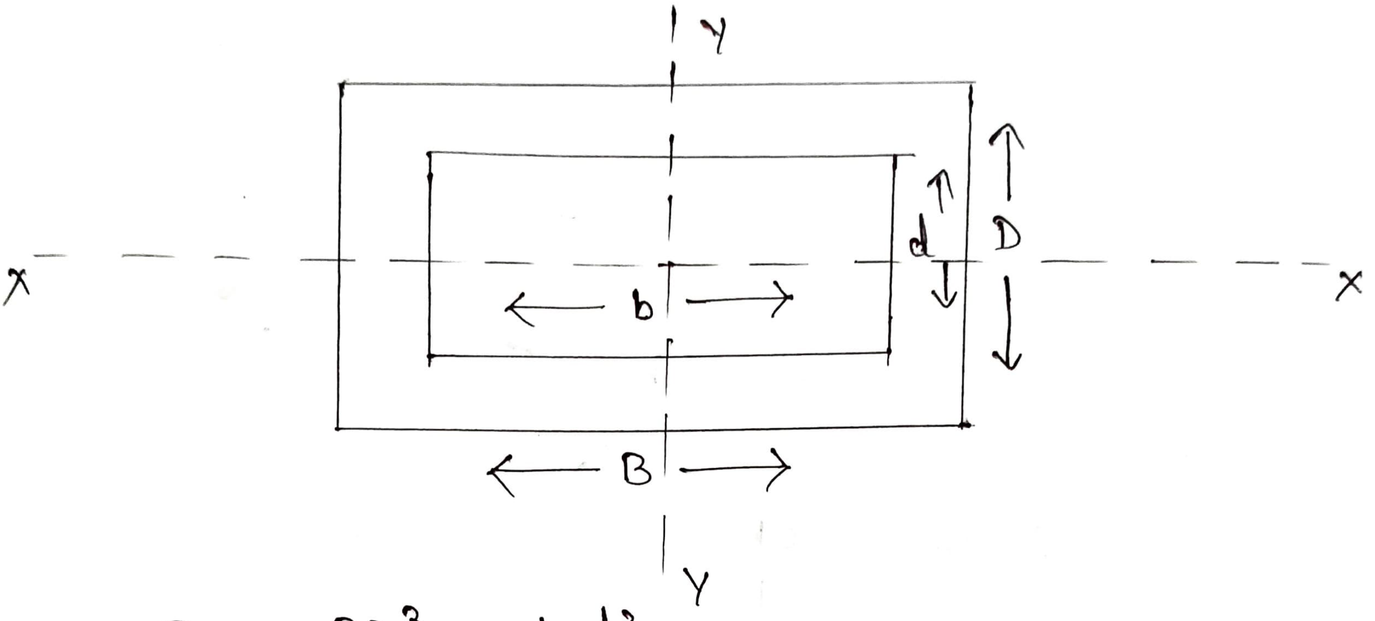
SQUARE SECTION :-

(ABOUT XX & YY AXIS)



$$\therefore I_{xx} = I_{yy} = \frac{a^4}{12}$$

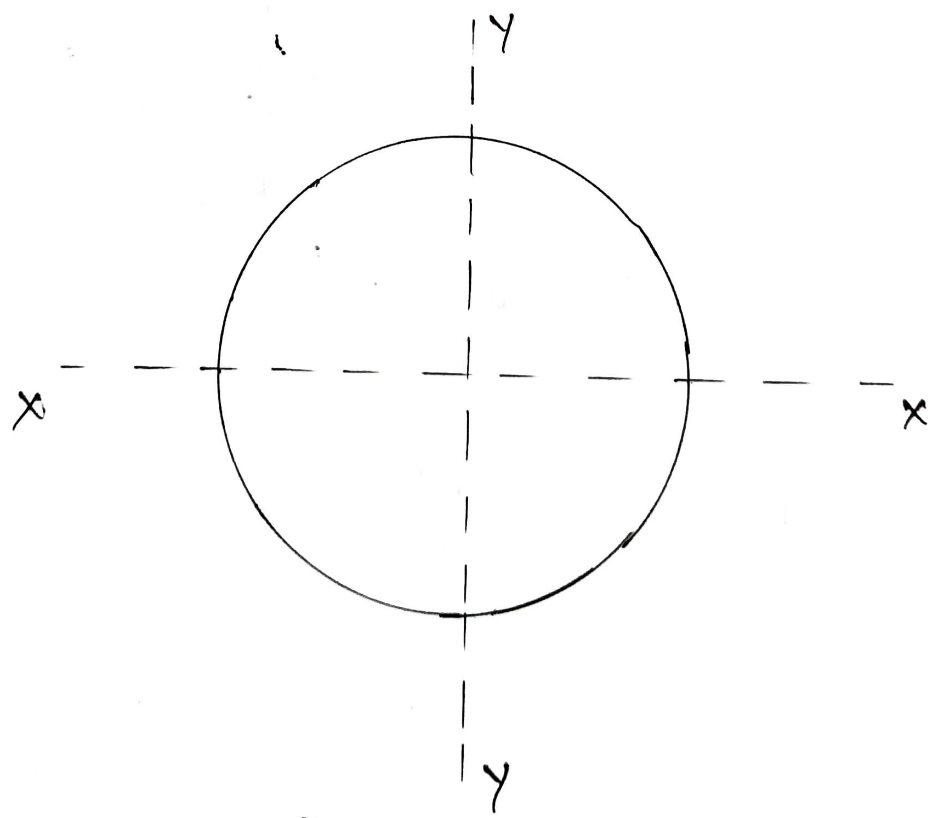
M.I OF A HOLLOW RECTANGULAR SECTION / HOLLOW SECTION (ABOUT XX AXIS)



$$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12}$$

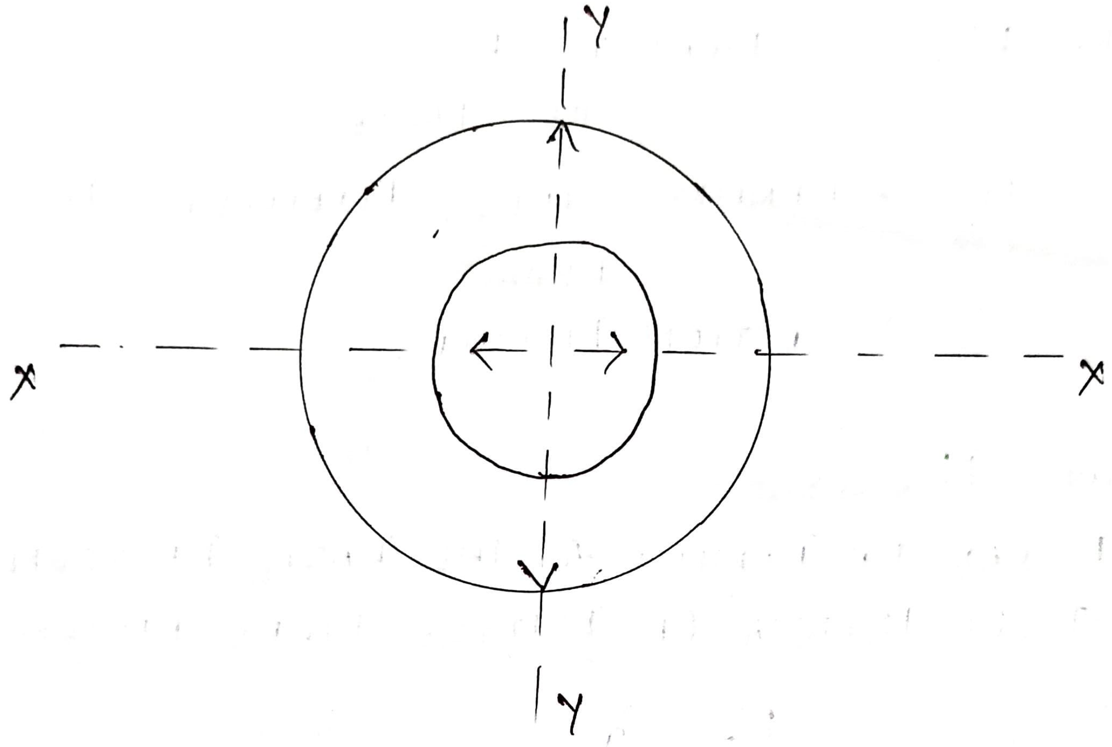
$$I_{yy} = \frac{DB^3}{12} - \frac{db^3}{12}$$

M.I OF A CIRCULAR SECTION :-



$$I_{xx} = I_{yy} = \frac{\pi D^4}{64}$$

M.I OF A HOLLOW CIRCULAR SECTION :-



$$I_{xx} = I_{yy} = \frac{\pi D^4}{64} - \frac{\pi d^4}{64}$$

MOMENT OF RESISTANCE (M.R) :-

MOMENT OF RESISTANCE IS DEFINED AS THE RESISTANCE TO MOMENT WHEN BENDING MOMENT IS APPLIED ON THE BEAM.

OR

IT CAN BE DEFINED AS THE PRODUCT OF BENDING STRESS & SECTION MODULUS.

$$\frac{\sigma_b}{y} = \frac{M}{I}$$

$$\Rightarrow \boxed{M = \sigma_b \left(\frac{I}{y}\right) = \sigma_b \cdot Z}$$

So $\boxed{M.R = \sigma_b . Z}$

WHERE MR → MOMENT OF RESISTANCE

σ_b → BENDING STRESS DEVELOPED ON BEAM

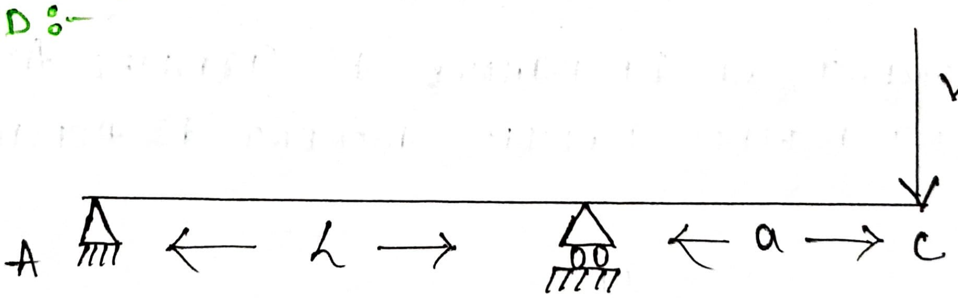
Z → SECTION MODULUS

SECTION MODULUS :-

IT CAN BE DEFINED AS THE RATIO BETWEEN THE MOMENT OF INERTIA OF EXTREME FIBRE DISTANCE (y)

So, $\boxed{Z = \frac{I}{y}}$

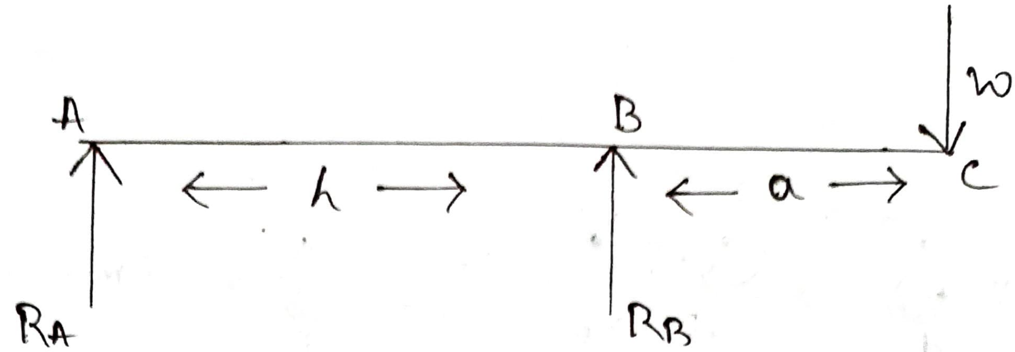
OVERHANGING BEAM SUBJECTED TO CONC. LOAD AT FREE END :-

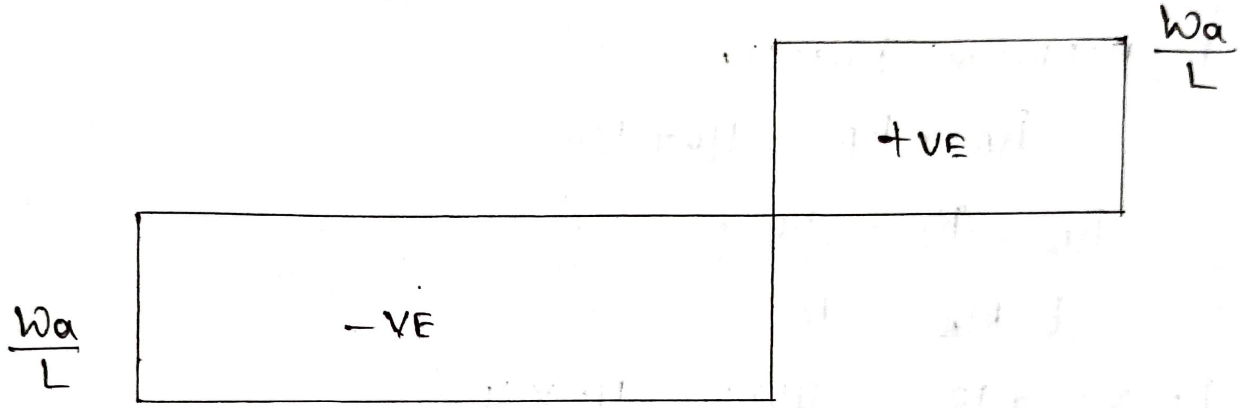


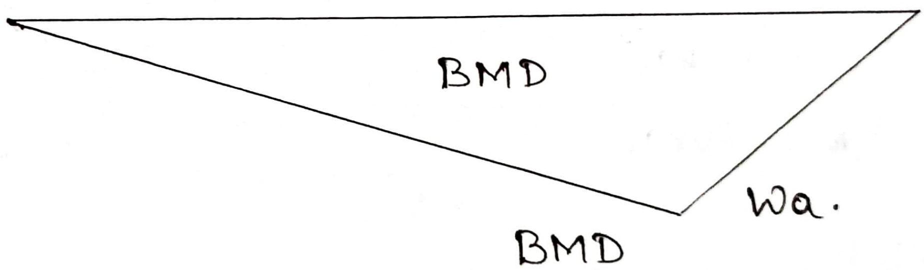
LET ABC BE THE OVERHANGING BEAM SUBJECTED TO A CONCENTRATED LOAD W AT FREE END AS SHOWN IN FIG.

SO TAKING MOMENT EQUILIBRIUM CONDITION ABOUT 'B' ; WE GET

$R_A l = Wa$





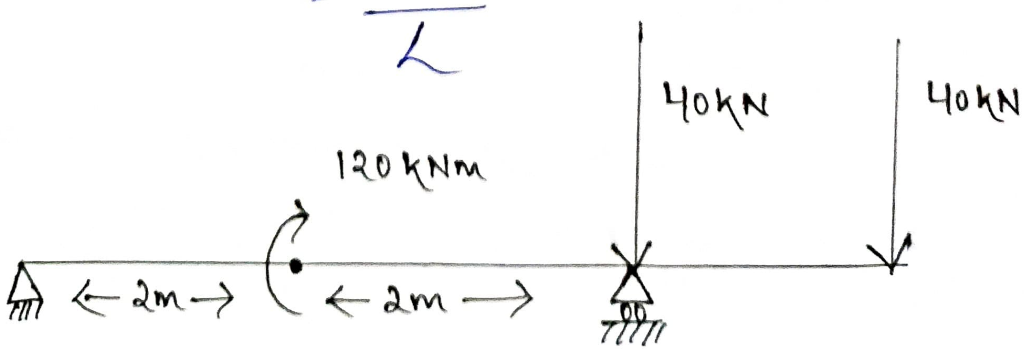


SECTION AB:-

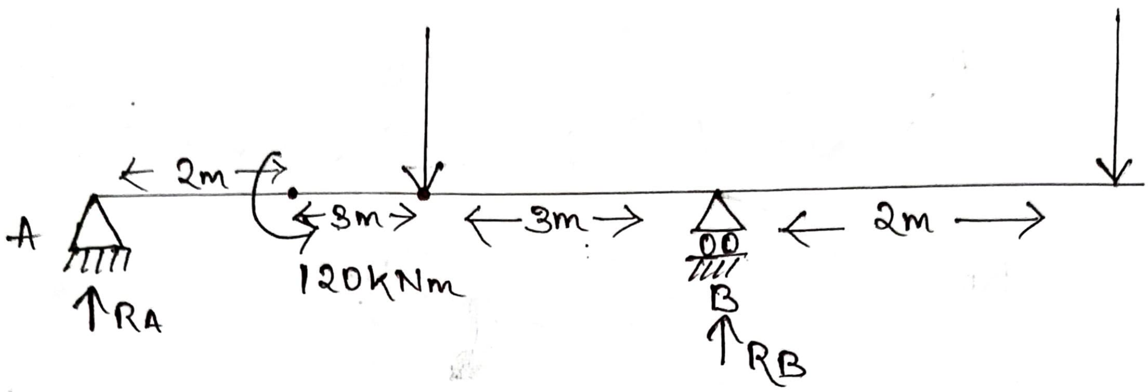
TAKE A SECTION XX FROM A;

So $SF_A = +R_A$

$= \frac{Wa}{L}$



OVERHANGING BEAM WITH COUPLE :-



EQUILIBRIUM EQN :-

$$R_A + R_B = 40 + 40$$

$$R_A + R_B = 80 \quad \text{——— (i)}$$

$$\Sigma M_A = 0$$

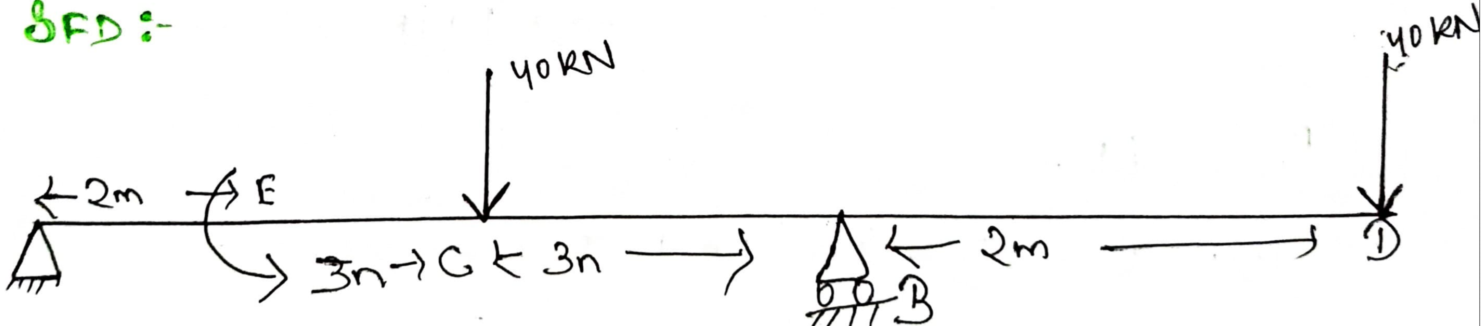
$$R_B \times 8 + 120 = 40 \times 5 + 40 \times 7$$

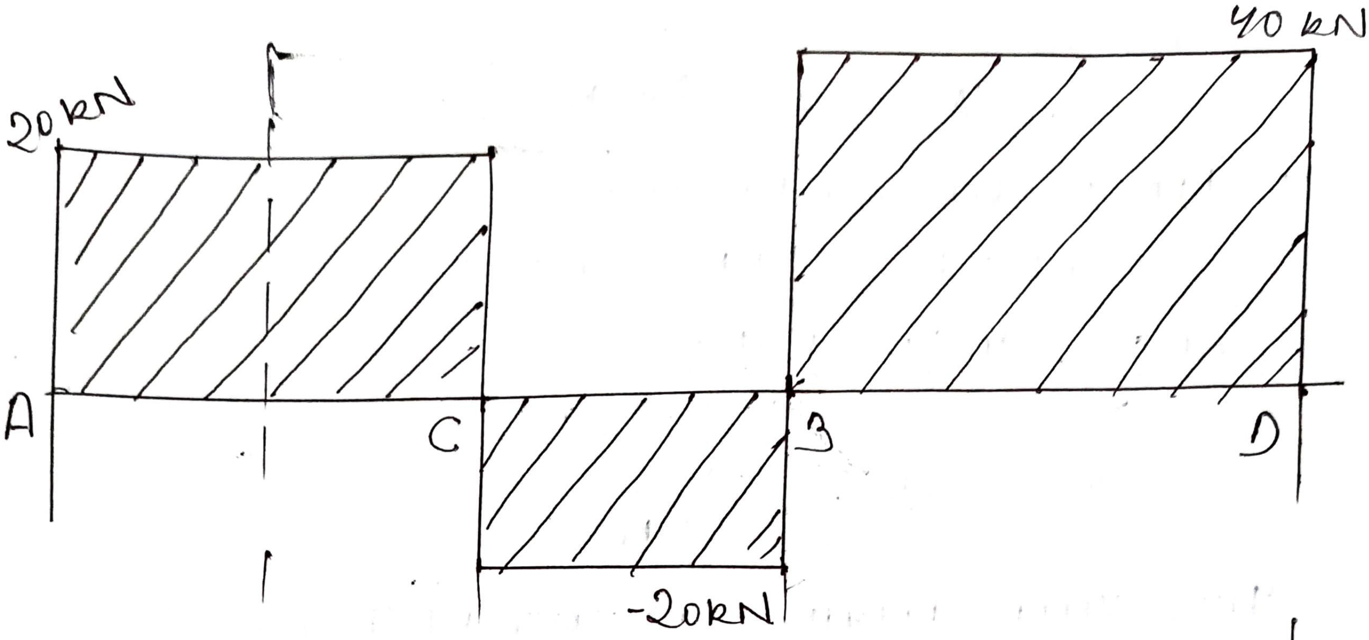
$$R_B \times 8 = 480$$

$$\Rightarrow R_B = 60KN$$

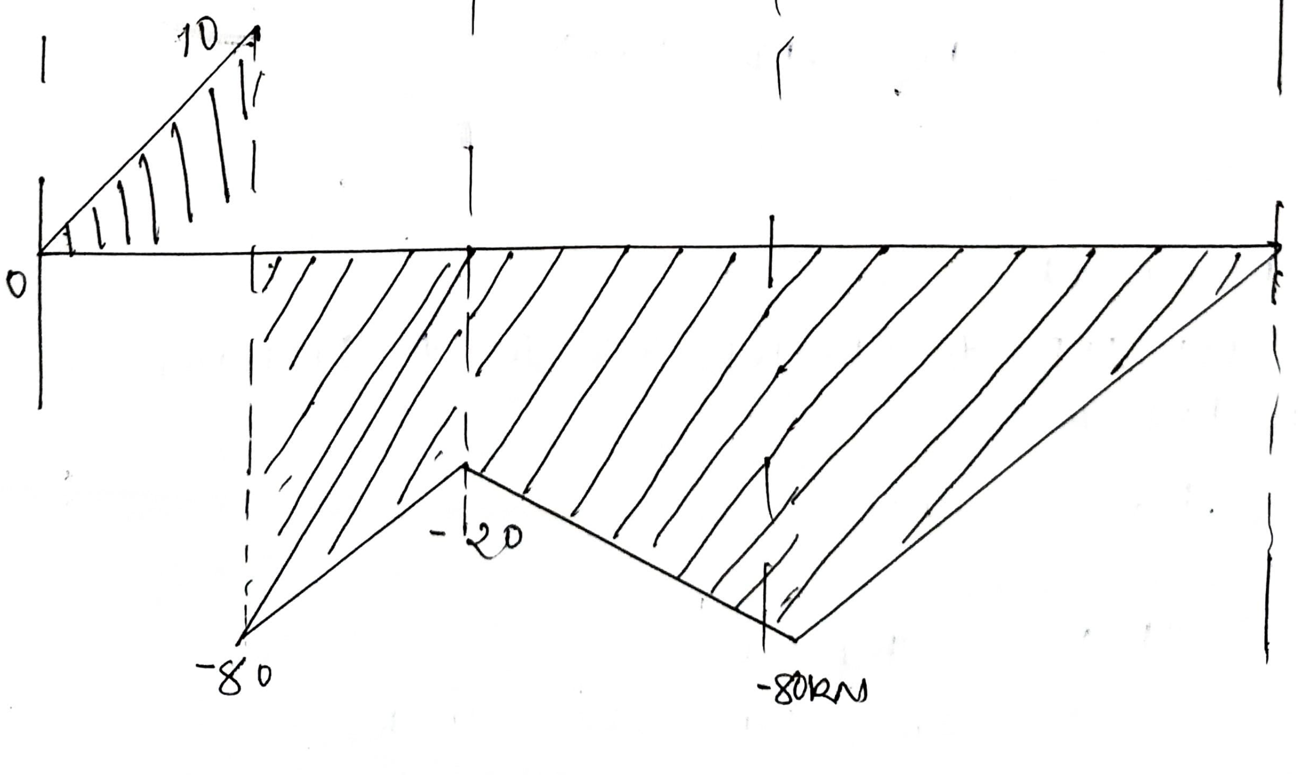
$$R_A = 20KN$$

SFD :-





40 kN
20 kN
A
C
B
D
-20 kN



10
0
-20
-80
-80 kN

SFD :-

SECTION AC :—

S.F at A = $+R_A$ = 20 kN

IT REMAINS CONSTANT UP TO C

$\therefore R_A = R_C = 20\,kN = S_A = S_C$

At C, 40 kN ACTING DOWNWARD

So $S_C = S_B = 20 - 40$
$= -20\,kN$

THIS VALUE REMAINS CONSTANT UP TO B.

SECTION BD :—

AT POINT B; R_B IS ACTING

So $S.F_B = +R_B - 20$
$= 60 - 20$
$= 40\,kN$

So SHEAR FORCE REMAINS CONSTANT UP TO D.

i.e. $SF_B = SF_D = 40\,kN$

BMD :-

SECTION AE :—

CONSIDER A SECTION XX AT A DISTANCE 'x' FROM END 'A' :—

$\therefore M_x = +R_A x$
$= 20x$

$\therefore$ At $x = 0$; $M_A = 0$

$x = 2m$; $M_E = 20 \times 2 = 40\,kN$

AT E ; 120 KNm IS ACTING ANTICLOCKWISE

SO NET MOMENT $= R_A \times 2 - 120$

$= 40 - 120$

$= -80$ KNM

SECTION EC :-

$M_x = + R_A (x+2) - 120$

$= 20(x+2) - 120$

$= 20x + 40 - 120$

$= 20x - 80$

At $x = 0$; $M_E = -80$ KNm

$x = 3$; $M_C = 60 - 80$

$= -20$ KNM

SECTION CB :-

$M_N = + R_A (x+5) - 40x - 120$

$= 20(x+5) - 40x - 120$

$= 100 + 20x - 40x - 120$

$= -20 - 20x$

At $x = 0$; $M_C = -20$ KNM

$x = 3$; $M_B = -20 - 60$

$= -80$ KNm

SECTION BD :-

$M_x = -40x$

$M_x = -40x$

$\therefore$ At $x = 0$; $M_D = 0$

$x = 2m$; $M_B = -40 \times 2 = -80$ KNm

Theory of column

Column:

- Any vertical member which can take axialy compressive load is called column or strey.
- The member able to take axialy compressive.
- The common features column and strut is both are subjected to compressive load.
- Short columns subjected to axial load.
- Let us consider a short column which is subjected to an axial compressive load as shown in the fig.
- The stress developed by the compressive.

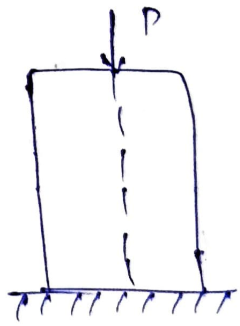
∴ crushing stress are compressive stress i.e.,

$$\sigma_c = P/A$$

Normal stress / permissiable stress = $\sigma = P/A$

∴ factor of safety = $\frac{\text{crushing stress}}{\text{permissiable stress}}$

- Factor of safety = $\frac{\sigma_c}{\sigma}$

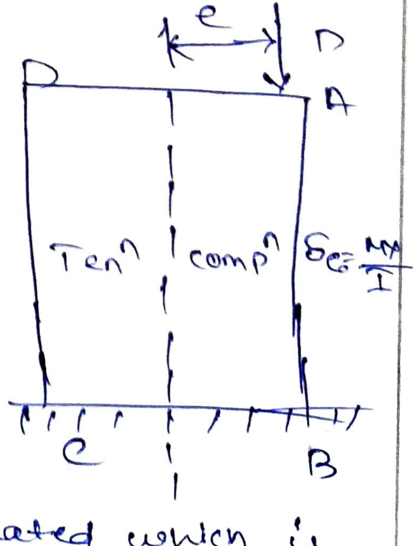


∴ safe load = $\frac{E_s}{E_c}$

$$\sigma_s = \frac{E_s}{E_c} \times \sigma_c$$

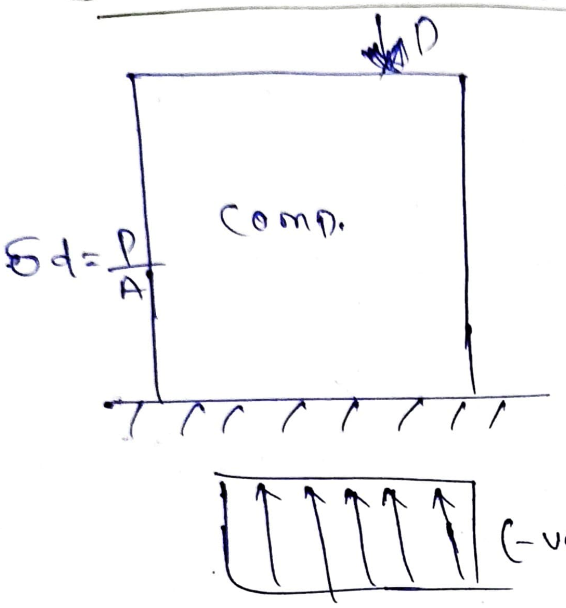
Eccentric load on column:

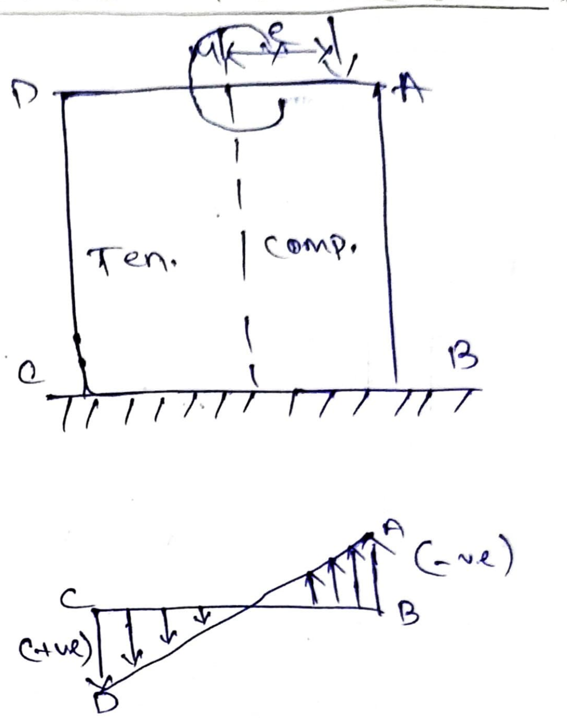
- The which is acting away from the axis of the column in known as eccentric load.
- Load eccentricity.
- It is the distance between the axis of the column and the point of Apln of the load.
- When on eccentric load acts on a column. it causes bending moment and due to the bending moment, bending stress is generated which is



compressive on one side and tensile on other side.

Stress distribution due to the eccentric load on the columns:





$\therefore \frac{\sigma_b}{y} = \frac{M}{I} \Rightarrow \sigma_b = \frac{My}{I}$

$\therefore \frac{\sigma(AB)}{\sigma(CD)}$ = comp (-ve) / Tension (+ve)

$\sigma_{(max)} AB = -\sigma_d - \sigma_b AB$

or $\rightarrow \sigma(max) AB = -(\sigma_d + \sigma_b AB)$

Nature is compressive

$\sigma(min)\, CD = -\sigma_d + \sigma_b(CD)$

(compression) (Tension)

Euler's Buckling load for long columns:

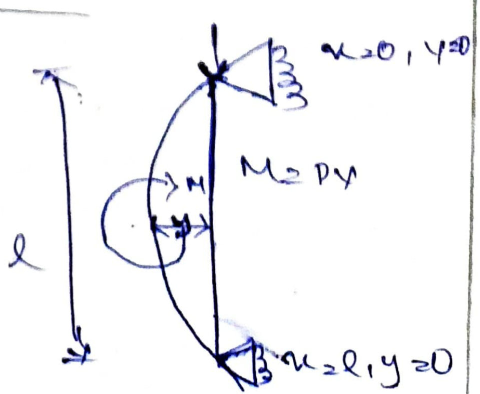
$M_x = EI \frac{d^2y}{dx^2}$

$\Rightarrow -Py = EI \frac{d^2y}{dx^2}$

$\Rightarrow EI \frac{d^2y}{dx^2} + Py = 0$

$\Rightarrow \frac{d^2y}{dx^2} + \frac{P}{EI} y = 0$

$\Rightarrow \frac{d^2y}{dx^2} + \lambda^2 y = 0$



THEORY OF COLUMNS.

Column:-

Vertical members of a building frame or any structural system which carry mainly axial compressive load are known as columns.

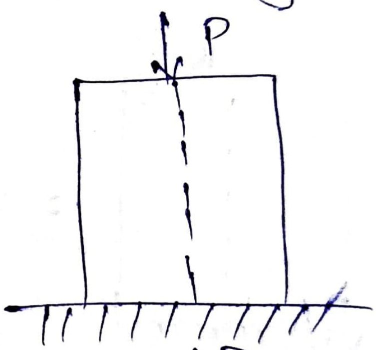
The Compression member of a truss is called truss.

The common feature of columns and struts that they are subjected to comp. forces.

Axial load:

Short columns S.T Axial load!

Let us consider a short column as shown in the figure subjected to an axial load (P).

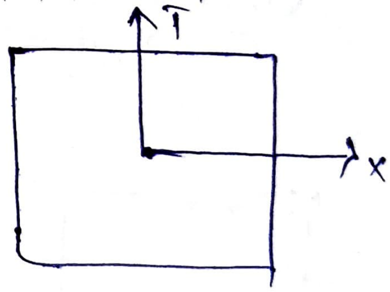


$$\text{Comp. stress}(\sigma) = \frac{P}{A}$$

A → Cross-sectional Area

If σ_c → crushing stress

Then σ_{per} → permissible comp. stress



$$\sigma_{per} = \frac{\sigma_{cr}}{F_{os}}$$

F_{os} = Factor of safety

Eccentric load on columns:

Eccentrically loaded masonary columns,

when eccentric load acts on a column,

It causes bending moment of produces tension on one side and comp. on other side.

Net stress at any point: stress due to axial load (comp)

+ stress due to bending.

$$= \sigma_{axial} + \sigma_b$$

$$= \frac{P}{A} + \frac{My}{I}$$

The resistance of masonary to tension is very little and for all practical purpose, it is taken as zero.

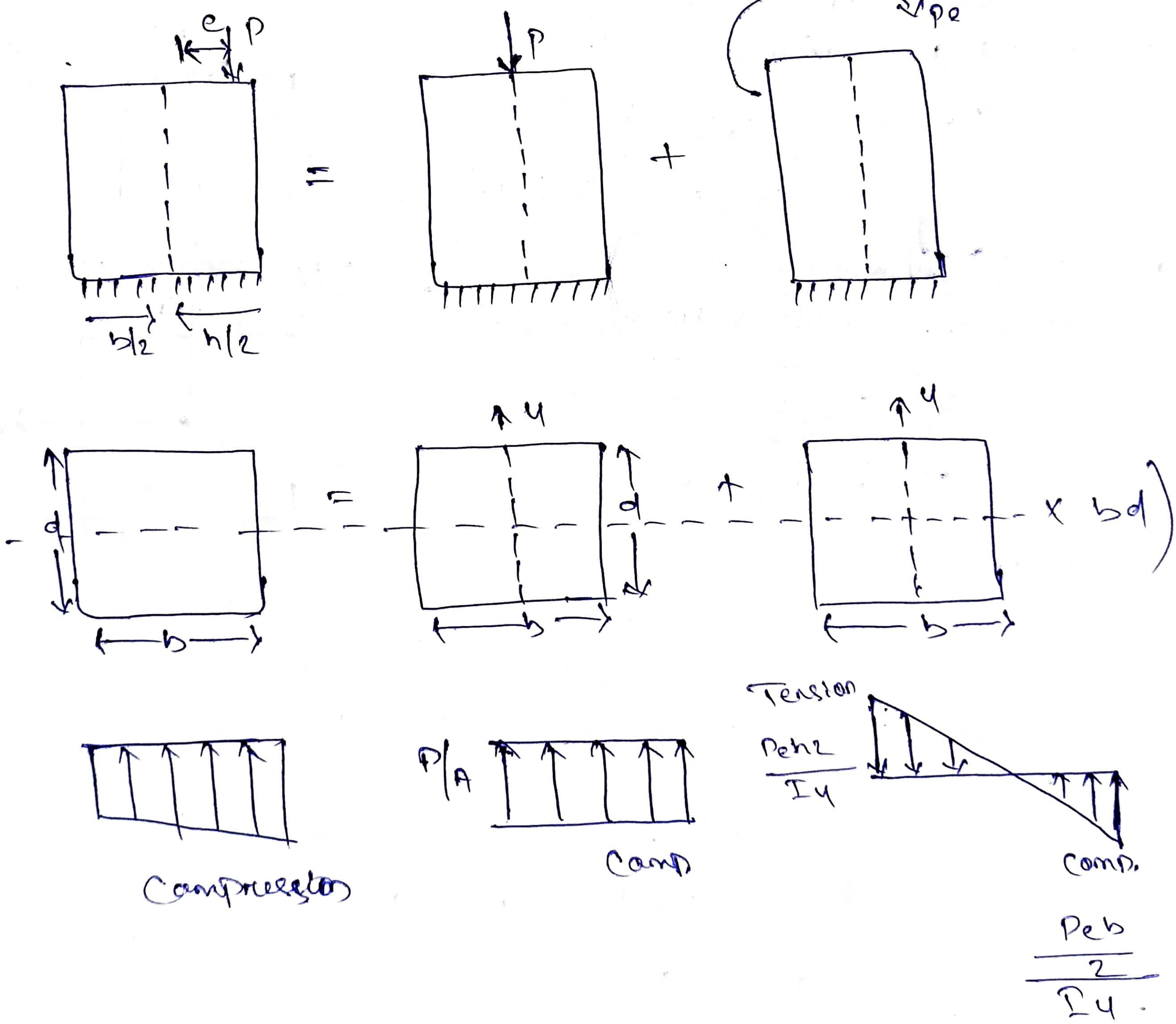
Hence in brick/stone masonary column, tension is neglected/not permitted.

Here we have 2 cases.

1. Eccentrically w.r.t only 1 axis

2. " " both axes.

Eccentrically w.r.t only 1 axis:



Maximum and minimum stresses:

$$\sigma_{min} = \frac{P}{A} - \frac{Pe\, b/2}{I_y}$$

$$\sigma_{max} = \frac{P}{A} + \frac{Pe\, b/2}{I_y}$$

$$I_y = \frac{1}{12} db^3 = \frac{1}{12} b^2 (A) \quad (\because A = bd)$$

Minimum stress:

$$\text{So } \sigma_{min} = \frac{P}{A} - \frac{Pe\, b/2}{\frac{1}{12} b^2 (A)}$$

$$= \frac{P}{A} - \frac{Peb}{2} \times \frac{12}{b^2 A}$$

$$= \frac{P}{A} - \frac{6Pe}{bA}$$

$$= \frac{P}{A} \cdot \left(1 - \frac{6e}{b}\right)$$

Direct stress (σ_d)

$$\sigma_d = \frac{P}{A}$$

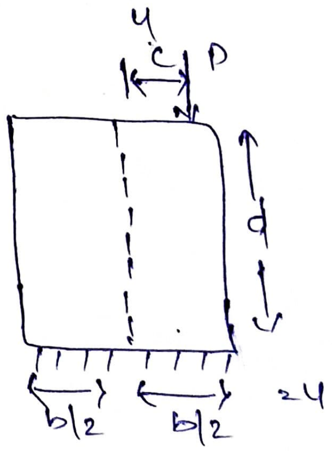
Bending stress (σ_b)

$$\sigma_b = \frac{MY}{I}$$

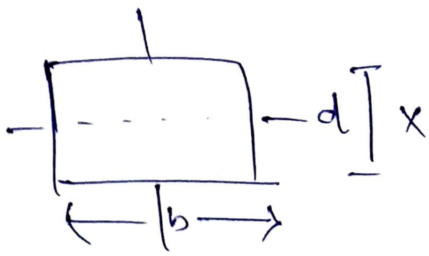
$$= \frac{(Pe)\left(\frac{b}{2}\right)}{\frac{1}{12} b^2 A}$$

$$= \frac{6Peb}{Ab}$$

$$= \frac{P}{A}\left(\frac{6e}{b}\right)$$



$(y = \frac{b}{2})$



Minimum stress!

$$\sigma_{max} = \frac{P}{A} + \frac{Pe\, b/2}{I_y}$$

$$= \frac{P}{A} + \frac{Pe\, b/2}{\frac{1}{12} b^2 A}$$

$$= \frac{P}{A} + \frac{Peb}{2} \times \frac{12}{b^2 A}$$

$$= \frac{P}{A} + \frac{P}{A}\left(\frac{6e}{b}\right)$$

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right)$$

Eccentricity $e = b/6$ for no tension

As $\sigma_{min} = 0$

$$\Rightarrow \frac{P}{A}\left(1 - \frac{6e}{b}\right) = 0$$

$$\Rightarrow \boxed{e = b/6}$$

$\therefore y = A\cos \lambda x + B\sin \lambda x.$ (roots)

Boundary conditions are at i) $x=0, y=0$

ii) $x=l, y=0$

at $x=0, y=0$

$A=0 \longrightarrow (i)$

at $x=l, y=0$

$0 = B\sin \lambda l$

'B' cannot be zero

$\sin \lambda l = 0$

$\Rightarrow \sin \lambda l = \sin n\pi$

$\Rightarrow \lambda l = n\pi$

$\Rightarrow \sqrt{\frac{P}{EI}} \cdot L = n\pi$

$\Rightarrow \left(\sqrt{\frac{P}{EI}}\right)^2 = \left(\frac{n\pi}{L}\right)^2$

$\Rightarrow \boxed{\frac{P}{EI} = \frac{n^2\pi^2}{L^2}}$

$\Rightarrow \boxed{P = \frac{n^2\pi^2 EI}{L^2}}$

→ According to different values of 'n' there are four different condition of the column

Case-1

→ Both ~~sides~~ ends are hinge

here $\boxed{n=1}$

$\therefore P = \frac{\pi^2 EI}{L^2}$

Case-2

→ One end is fix and other end in free

→ Here $\boxed{n=\frac{1}{2}}$

$\therefore \boxed{P = \frac{\pi^2 EI}{4L^2}}$

Case-3

→ Both the ends are fix

→ $\boxed{n=2}$

$$P = \frac{4\pi^2 EI}{L^2}$$

Case-4

→ one end is fix another end is hinge

→ Here $\boxed{n=\sqrt{2}}$

$$P = \frac{2\pi^2 EI}{L^2}$$

Calculation:

Case-I:

$$P = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 EL^2}{Le^2}$$

Case-2

$$P = \frac{\pi^2 EI}{4L^2} = \frac{\pi^2 EI}{Le^2}$$

$$\therefore Le^2 = 4L^2 \Rightarrow Le = 2L$$

$$= \frac{n^2\pi^2 EI}{L^2}$$

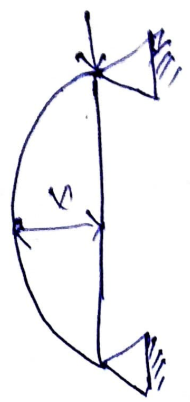
$$\Rightarrow n^2 = \frac{1}{4} \Rightarrow n = \frac{1}{2}$$

case-3

$$P = \frac{4\pi^2 EI}{L^2} = \frac{\pi^2 EI}{Le^2}$$

$$\Rightarrow Le^2 = L^2/4$$

Slenderness ratio:



$$P = \frac{\pi^2 EI}{Le^2}$$

$$= \frac{\pi^2 E(Ak^2)}{Le^2}$$

$$= \frac{\pi^2 EA}{(Le/k)^2} = \frac{\pi^2 EA}{\lambda^2}$$

$$\lambda^2 = \frac{\pi^2 EA}{P} \Rightarrow \lambda = \sqrt{\frac{\pi^2 EA}{P}} \Rightarrow \lambda = \sqrt{\frac{\pi^2 EA}{\sigma A}}$$

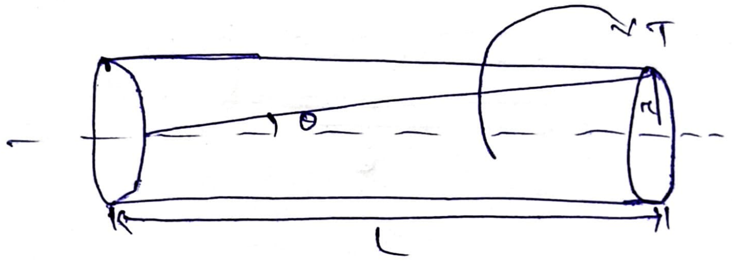
$$\Rightarrow \lambda = \sqrt{\frac{\pi^2 E}{\sigma}}$$

Torsion

Introduction:

A member is said to be in pure torsion when its cross section is subjected to torsional moment and not accompanied by any bending moment and axial force.

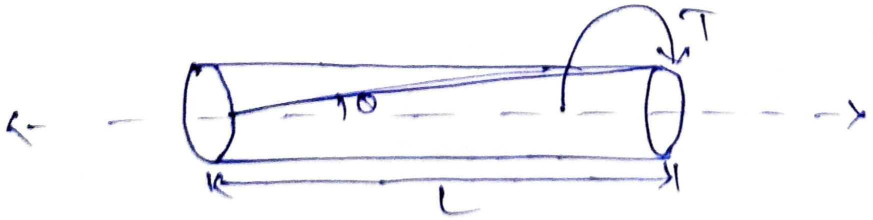
→ Due to the torsional moment the torsion stress (τ) is developed in the material and the shear stress is defined as internal resistance set up within the material because of the torsional moment.



Assumption in pure torsion:

→ The material in homogenious and isotropic
→ The stress are within the elastic limit it means stress is directly proportional to strain.
→ The twisting along the length of the shaft remains uniform.
→ cross section of the shaft which are place before twisting remains plane after twisting.

Torsion equation for a shaft:



$$\frac{T}{I_P} = \frac{\tau}{r} = \frac{G\theta}{L}$$

where,
T = twisting moment applied on the solid shaft (NM)

I_P = polar moment of inertia of the shaft having cross sectional section $I_P = 2 \times I$ (mm^4, m^4)

τ = Shear stress developed on the solid shaft because of twisting moment.

r = is the radius of circular cross section (m, mm)

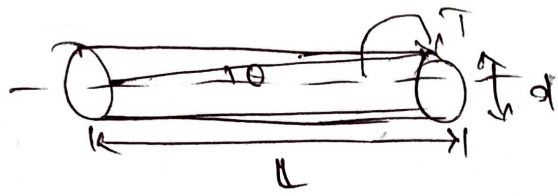
G = Modulus of rigidity (N/m^2, N/mm^2)

θ = Angle of twist in radian.

1 degree $\longrightarrow$ $\pi/180$ Rad.

L = length of the shaft.

Expression for twisting moment (T) of solid shaft!



$$T/I_P = \tau/r$$

$$\Rightarrow \frac{T}{I_P} = \tau/r$$

$$\Rightarrow T = \frac{\tau I_P}{r}$$

$$\Rightarrow T = \frac{\tau 2I}{r} \Rightarrow T = \frac{\tau \times \frac{\pi d^4}{64} \times 2}{r}$$

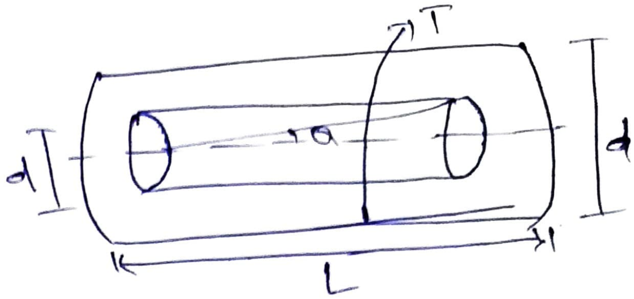
$$\Rightarrow T = \frac{\tau \pi d^4/32}{\frac{d}{2}}$$

$$\Rightarrow T = \frac{\tau \pi d^4}{32} \times \frac{2}{d}$$

$$\Rightarrow T = \frac{\tau \pi d^3}{16} \qquad \Rightarrow \boxed{\tau = \frac{16T}{\pi d^3}}$$

Expression for twisting moment (T) of Hollow shaft

$$\frac{T}{I_p} = \frac{\tau}{r}$$



$$\Rightarrow T = \frac{\tau I_p}{r} \Rightarrow T = \frac{\tau \cdot 2 \cdot I}{\left(\frac{D}{2}\right)}$$

$$T = \tau \frac{\pi}{64} (D^4 - d^4) \times \frac{2}{D} \times 2$$

$$\Rightarrow T = \tau \frac{\pi}{16D} (D^4 - d^4)$$

$$\Rightarrow T = \tau \times \frac{\pi}{16D} \times D^4 \left\{1 - (d/D)^4\right\}$$

$$\Rightarrow \boxed{T = \tau \times \frac{\pi}{16} D^3 (1 - k^4)}$$

Torsional rigidity / Torsional stiffness / stiffness of shaft:

$$\frac{T}{I_p} = \frac{G\theta}{L}$$

$$\boxed{\frac{TL}{\theta} = G I_p}$$

∴ Power of a shaft

$$\therefore \boxed{P = \frac{2\pi N T}{60}}$$

$$\therefore \boxed{T = \frac{60 P}{2\pi N}}$$

$$\boxed{\tau = \frac{16T}{\pi d^3}}$$